Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank

Xavier Freixas, Bruno Parigi and Jean-Charles Rochet

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ABSTRACT

We model systemic risk in an interbank market. Banks face liquidity needs as consumers are uncertain about where they need to consume. Interbank credit lines allow to cope with these liquidity shocks while reducing the cost of maintaining reserves. However, the interbank market exposes the system to a coordination failure (gridlock equilibrium) even if all banks are solvent. When one bank is insolvent, the stability of the banking system is affected in various ways depending on the patterns of payments across locations. We investigate the ability of the banking system to withstand the insolvency of one bank and whether the closure of one bank generates a chain reaction on the rest of the system. We analyze the coordinating role of the Central Bank in preventing payments systemic repercussions and we examine the justification of the Too-big-to-fail-policy.

Keywords: systemic risk; contagion; interbank markets

JEL Codes: E58, G21

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1 INTRODUCTION

The possibility of a systemic crisis affecting the major financial markets has raised regulatory concern all over the world. Whatever the origin of a financial crisis, it is the responsibility of the regulatory body to provide adequate fire walls for the crisis not to spill over other institutions. In this paper we explore the possibilities of contagion from one institution to another that can stem from the existence of a network of financial contracts. These contracts are essentially generated from three types of operations: the payments system, the interbank market and the market for derivatives.[1] Since these contracts are essential to the financial intermediaries' function of providing liquidity and risk sharing to their clients, the regulating authorities have to set patterns for Central Bank intervention when confronted with a systemic shock. In recent years, the 1987 stock market crash, the Saving and Loans crisis, the Mexican, Asian and Russian crises and the crisis of the Long Term Capital Management hedge fund have all shown the importance of the intervention of the Central Banks and of the international financial institutions in affecting the extent, contagion, patterns and consequences of the crises.[2]

In contrast to the importance of these issues, theory has not succeeded yet in providing a convenient framework to analyze systemic risk so as to derive how the interbank markets and the payments system should be structured and what the Lender of Last Resort (LOLR) role should be.

A good illustration of the wedge between theory and reality is provided by the deposits shift that followed

[1] There is ample empirical evidence on financial contagion. For a survey see de Bandt and Hartmann (1998). Kaufman (1994) reviews empirical studies that measure the adverse effects on banks' equity returns of default of a major bank and of a sovereign borrower or unexpected increases in loan-loss provisions announced by major banks. Others have studied contagion through the flow of deposits (Saunders and Wilson 1996), and using historical data (Gorton 1988, Schoenmaker 1996, and Calomiris and Mason 1997). Whatever the methodology, these studies support the view that pure panic contagion is rare. Far more common is contagion through perceived correlations in bank asset returns (particularly among banks of similar size and/or geographical location).

[2] A well known episode of near financial gridlock where a coordinating role was plaid by the Central Bank is represented by the series of events the day after the stock crash of 1987. Brimmer (1989 pp.14-15) writes that "On the morning of October 20, 1987, when stock and commodity markets opened, dozens of brokerage firms and their banks had extended credit on behalf of customers to meet margin calls, and they had not received balancing payments through the clearing and settlement systems. [...] As margin calls mounted, money center banks (especially those in New York, Chicago, and San Francisco) were faced with greatly increased demand for loans by securities firms. With an eye on their capital ratios and given their diminished taste for risk, a number of these banks became increasingly reluctant to lend, even to clearly creditworthy individual investors and brokerage firms.[...] To forestall a freeze in the clearing and settlement systems, Federal Reserve officials (particularly those from the Board and the Federal Reserve Bank of New York) urged key money center banks to maintain and to expand loans to their creditworthy brokerage firm customers."

the distress of Bank of Credit and Commerce International (BCCI). In July 1991, the closure of BCCI in the UK made depositors with smaller banks switch their funds to the safe haven of the big banks, the so-called ”flight to quality” (Reid 1991). Theoretically this should not have had any effect, because big banks should have immediately lent again these funds in the interbank market and the small banks could have borrowed them. Yet the reality was different: the Bank of England had to step in, to encourage the large clearers to help those hit by the trend. Some packages had to be agreed (as the £200m. to the National Home Loans mortgage lender), thus supplementing the failing invisible hand of the market. So far theory has not been able to explain why the intervention of the LOLR in this type of events was important.

Our motivation to analyze a model of systemic risk stems from both the lack of a theoretical set up, and the lack of consensus on the way the LOLR should intervene. In this paper we analyze interbank networks, focusing on possible liquidity shortages and on the coordinating role of the Financial Authorities - which we refer to as the Central Bank for short – in avoiding and solving them. To do so we construct a model of the payment flows that allows us to capture in a simple fashion the propagation of financial crises in an environment where both liquidity shocks and solvency shocks affect financial intermediaries that fund long term investments with demand deposits.

We introduce liquidity demand endogenously by assuming that depositors are uncertain about where they have to consume. This provides the need for a payments system or an interbank market.[3] In this way we extend the model of Freixas and Parigi (1998) to more than two banks, to different specifications of travel patterns and consumers' preferences. The focus of the two papers is different. Freixas and Parigi consider the trade-off between gross and net payments systems. In the current paper we concentrate instead on system-wide financial fragility and Central Bank policy issues. This paper is also related to Freeman (1996a,b). In Freeman, demand for liquidity is driven by the mismatch between supply and demand of goods by spatially separated agents that want to consume the good of the other location, at different times. If agents' travel patterns are not perfectly synchronized, a centrally accessible institution (e.g. a clearing house) may arise to provide means of payments. This allows to clear the debt issued by

[3] Payment needs arising from agents' spatial separation with limited commitment and default possibilities were first analyzed in Townsend (1987). For the main theoretical issues related to systemic risk in payment systems see Berger, Hancock and Marquardt (1996) and Flannery (1996), for an analysis of peer monitoring on the interbank market see Rochet and Tirole (1996) and for an analysis of the main institutional aspects see Summers (1994).

the agents to back their demand. In our paper, instead, liquidity demand arises from the strategies of agents with respect to the coordination of their actions.

Our main findings are, first, that, under normal conditions, a system of interbank credit lines reduces the cost of holding liquid assets. However, the combination of interbank credit and the payments system make the banking system prone to experience (speculative) gridlocks, even if all banks are solvent. If the depositors in one location wishing to consume in other locations believe that there will not be enough resources for their consumption at the location of destination, their best response is to withdraw their deposits at the home location. This triggers the early liquidation of the investment at the home location, which, by backward induction, makes it optimal for the depositors in other locations to do the same.

Second, the structure of financial flows affects the stability of the banking system with respect to solvency shocks. On the one hand, interbank connections enhance the ”resiliency” of the system to withstand the insolvency of a particular bank, because a proportion of the losses on one bank's portfolio is transferred to other banks through the interbank agreements. On the other hand, this network of cross liabilities may allow an insolvent bank to continue operating through the implicit subsidy generated by the interbank credit lines, thus weakening the incentives to close inefficient banks.

Third, the Central Bank has a role to play as a ”crisis manager”. When all banks are solvent, the Central Banks's role to prevent a speculative gridlock is simply to act as a coordinating device. By guaranteeing the credit lines of all banks, the Central Bank eliminates any incentive for early liquidation. This entails no cost for the Central Bank since its guarantees are never used in equilibrium. When instead one bank is insolvent because of poor returns on its investment, the Central Bank has a role in the orderly closure of this bank. When a bank is to be liquidated, the Central Bank has to organize the bypass of this defaulting bank in the payment network and provide liquidity to the banks that depend on this defaulting bank. Furthermore, since the interbank market may loosen market discipline, there is a role for supervision with the regulatory agency having the right to close down a bank even if this bank is not confronted with any liquidity problem.

Fourth, when depositors have asymmetric payments needs across space, the role of the locations where many depositors want to access their wealth (money center locations) becomes crucial for the stability of the entire banking system. We characterize the too-big-to-fail (TBTF) approach often followed by Central Banks in dealing with the financial distress of money center banks, i.e. banks occupying key positions in the interbank network system.

The results of our paper are closely related to those of Allen and Gale (1998) where financial connections arise endogenously between banks located in different regions. In our work inter-regional financial connections arise because depositors face uncertainty about the location where they need to consume. In Allen and Gale, instead, financial connections arise as a form of insurance: when liquidity preference shocks are imperfectly correlated across regions, cross holdings of deposits by banks redistribute the liquidity in the economy. These links, however, expose the system to the possibility that a small liquidity shock in one location spread to the rest of the economy. Despite the apparent similarities between the two models and the related conclusions pointing at the relevance of the structure of financial flows, it is worth noticing that in our paper instead we focus on the implications for the stability of the system when one bank may be insolvent.

This paper is organized as follows. In section 2 we set up our basic model of an interbank network. In section 3 we describe the coordination problems that may arise even when all banks are solvent. In Section 4 we analyze the ”resiliency” of the system when one bank is insolvent. In Section 5 we investigate whether the closure of one bank triggers the liquidation of others, and we show under which conditions the intervention of the Central Bank is needed to prevent a domino or contagion effect. Section 6 provides an example of asymmetric travel patterns and its implications for Central Bank intervention. Section 7 discusses the policy implications, offers some concluding remarks and points to possible extensions.

2 THE MODEL

2.1 **Basic Set Up**

We consider an economy with 1 good and N locations with exactly one bank[4] in each location. There is a continuum of risk-neutral consumers of equal mass (normalized to one) in each location. There are three periods: $t = 0, 1, 2$. The good can be either stored from one period to the next or invested. Each consumer is endowed with one unit of the good at $t = 0$. Consumers cannot invest directly but must deposit their endowment in the bank of their location, which stores it or invests it for future consumption. Consumption takes place at $t = 2$ only. The storage technology yields the riskless interest rate which we normalize at 0. The investment of bank i yields a gross return R_i at $t = 2$, for each unit invested at $t = 0$, and not liquidated at $t = 1$. At $t = 0$ the bank optimally chooses the fraction of deposits to store or invest. The deposits contract specifies the amount c_1 received by depositors if they withdraw at $t = 1$, and their bank is solvent. At $t = 2$, remaining depositors equally share the returns of the remaining assets. To finance withdrawals at $t = 1$ the bank uses the stored good, and for the part in excess, liquidates a fraction of the investment. Each unit of investment liquidated at $t = 1$ gives only α units of the good (with $\alpha \leq 1$).

We extend this model by introducing a spatial dimension: a fraction $\lambda > 0$ of the depositors (we call them the *travelers*) must consume at $t = 2$ in other locations. The remaining $(1 - \lambda)$ depositors (the *non travelers)* consume at $t = 2$ in the home location. So in our model, consumers are uncertain about *where* they need to consume.

Our model is in the spirit of Diamond and Dybvig's (1983) (hereafter D-D) but with a different interpretation. In D-D, risk averse consumers are subject to a preference shock as to *when* they need to consume. The bank provides insurance by allowing them to withdraw at $t = 1$ but exposes itself to the risk of bank runs since it funds an illiquid investment with demand deposits. Our model corresponds to a simplified version of D-D where the patient consumers must consume at home or in the other location(s) and the

[4] This unique bank can be interpreted as a mutual bank, in the sense that it does not have any capital and acts in the best interest of its customers.

proportion of impatient consumers is arbitrarily small. This allows us to concentrate on the issue of payments across locations without analyzing intertemporal insurance. Our focus is on the coordination of the consumers of the various locations, and not on the time-coordination of the consumers at the same location.[5]

Since we analyze interbank credit, the good should be interpreted as cash (i.e. Central Bank money). Cash is a liability of the Central Bank that can be moved at no cost, but only by the Central Bank.[6]

If we interpret our model in terms of payment systems the sequence of events takes place within a 24-hour period. Then we could interpret $t = 0$ as the beginning of the day, $t = 1$ as intraday, $t = 2$ as overnight, and the liquidation cost $(1-\alpha)$ as the cost of (fire) selling monetary instruments in an illiquid intraday market.[7]

We assume that R_i is publicly observable at $t = 1$. In a multi-period version of our model, R_i would be interpreted as a signal on bank i's solvency that could provoke withdrawals by depositors or liquidation by the central bank at $t = 1$ (intraday). For simplicity, we adopt a two period model, and we assume here that the bank is liquidated anyway, either at $t = 1$, or at $t = 2$. Notice that even if R_i is publicly observed at $t = 1$ (we make this assumption to abstract from asymmetric information problems) it is not verifiable by a third party at $t = 1$ (only ex-post, at $t = 2$). Therefore the deposit contract cannot be fully conditioned on R_i. More specifically, the amount c_1 received for a withdrawal at $t = 1$ can just depend on the only verifiable information at $t = 1$, namely the closure decision. We denote by D_0 this contractual amount[8] in the case where the bank is not closed at $t = 1$. On the other hand, whenever the

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[5] The demandable deposit feature of the contract in this model does not rely necessarily on intertemporal insurance but may have alternative rationales. For example Calomiris and Kahn (1991) suggest that the right to withdraw on demand, accompanied by a sequential service constraint, gives informed depositors a credible threat in case of misuse of funds by the bank.

[6] Models in the tradition of Diamond-Dybvig have typically left the characteristics of the one good in the economy in the mist. This is all right in a microeconomic set-up, but the model has monetary implications that lead to a different interpretation depending on the fact that the good is money or not. In particular, if the good is not money, but for example wheat, then Wallace (1988)'s criticism applies. In other words, if the good was interpreted as wheat we would have to justify why the Central Bank was endowed with a superior transportation technology. As we assume the good to be money, it is the fact that commercial banks use Central Bank money to settle their transactions that gives the Central Bank the monopoly of issuing cash. Therefore the possibility to transfer money from one location to another corresponds to the ability to create and destroy money. Notice, also that interpreting the good as cash implies that currency crises, which are often associated with systemic risk, are left out of our analysis. This is so because "cash" is then limited by the level of reserves of the Central Bank.

[7] Since banks specialize in lending to information-sensitive customers, 1-α can also be interpreted as the cost of selling loans in the presence of lemons problem.

[8] This amount results from ex-ante optimal contracting decisions that could be solved explicitly. For conciseness, we take D_0

bank is closed (whether at $t = 1$ or at $t = 2$) its depositors equally share its assets (see Assumptions 1 and 2 below).

In order to be more explicit, it is worth examining the characteristics of the optimal deposit contract in the D-D model when the proportion of early diers tends to zero. This provides a useful benchmark for measuring the exposure of the interbank system to market discipline in our multi-bank model. Let μ denote the proportion of early diers and u be the Von Neumann Morgenstern utility function of depositors, with $u' > 0$ and $u'' < 0$. The optimal deposit contract (c_1, c_2) maximizes $\mu u(c_1) + (1-\mu)u(c_2)$ under the constraint $\mu c_1 + (1-\mu)c_2/R = 1$. Together with the budget constraint, this optimal contract is characterized by the first order condition:

$$u'(c_1) = Ru'(c_2). \tag{1}$$

When μ tends to zero, it is easy to see that c_2 tends to R Since $R > 1$ and u' is decreasing, we see immediately that $D_0 < R$. Therefore if the bank is known to be solvent no depositor has interest to withdraw unilaterally before he or she actually needs the money.

2.2 General formulation of consumption across space

Travel patterns, that is which depositor travels and to which location, are exogenously determined by nature at $t = 1$ and privately revealed to each depositor. They result from depositors' payment needs arising from other aspects of their economic activities. For each depositor initially at location i, nature determines whether he or she travels and in which location j he or she will consume at $t = 2$.[9] To consume at $t = 2$ at location j $(i \neq j)$ the travelers at location i can withdraw at $t = 1$ and carry the cash by themselves from location i to location j. The implicit cost of transferring the cash across space is the foregone investment return.[10] This motivates the introduction of credit lines between banks to minimize

as given. Notice that if R_i was verifiable, D_0 could be contingent on it and the risk of contagion could be fully eliminated.

[9] More generally, depositors receive shocks to their preferences which determine their demand for the good indexed by a particular location.

[10] We could also add an explicit cost of "travelling with the cash" (i.e. bypassing the payments system). It would not affect our results.

the amount of good not invested. The credit line granted by bank j to bank i gives the depositors of bank i going to bank j the right to have their deposits transferred to location j and obtain their consumption at $t = 2$ as a share of the assets at bank j at date $t = 2$.

A way to visualize the credit line granted by bank j to bank i, is to think that consumers located at i arrive in location j at $t = 2$ with a check written on bank i and credited in an account at bank j. Bank i, in turn, gives credit lines to one or more banks as specified below.[11] At $t = 2$ the banks compensate their claims and transfer the corresponding amount of the good across space. The technology to transfer the good at $t = 2$ is available for trades between banks only.

To make explicit the values of the assets and the liabilities resulting from interbank relations we adopt the simplest sharing rule, namely:

Assumption 1. *All the liabilities of a bank have the same priority at t=2.*

This rule defines how to divide bank's assets at $t = 2$ among the claim holders. It implies that credit lines are honored in proportion to the amount of the assets of the bank at date $t = 2$. In particular if D_i is the ex post value of a (unit of) deposit in bank i, then $D_i = \frac{Bank_i\ Total\ Assets}{Bank_i\ Total\ Liabilities}$. This assumption implies also that the banks cannot determine the location of origin of the depositors; thus depositors become anonymous and the banks cannot discriminate among them. Notice that more complex priority rules could be more efficient in the resolution of liquidity crises. However, we assume that they are not feasible in our context: this is a reduced form assumption aiming at capturing the limitations of the information that is available in interbank networks. An additional assumption is needed to describe what happens in case a bank is closed at $t = 1$.

Assumption 2. *If a bank is closed at time* 1 *its assets are shared between its own depositors only.*

Assumption 2 simply means that when the bank is closed at time $t = 1$, only its depositors have a claim on its assets. Bank closure at time 1 may come from a decision of the regulator, or from the withdrawals of all depositors. Assumption 2 implies that when a bank is closed at time 1, it is deleted from the interbank network.

[11] For a similar characterization of credit chains in the context of trading arrangements, see Kiyotaki and Moore (1997).

Let π_{ij} be the measure of depositors from location i consuming at location j, where i can take any value including j, and let t_{ij} be the proportion of travelers going from location i to j, $j \neq i$ (by definition, $t_{ii} = 0$). The matrix Π that defines where consumers go and in which proportions is related to the matrix T of travel patterns by:

$$\Pi \equiv (1-\lambda) I + \lambda T \tag{2}$$

where $\Pi = (\pi_{ij})_{ij}$, I is the identity matrix, and $T = (t_{ij})_{ij}$. This specification allows us to parameterize independently two features of the payment system: λ captures the intensity of interbank flows and the matrix T captures the structure of these flows. By definition, we have for all i, $\sum_j \pi_{ij} = 1$. For the sake of simplicity, unless otherwise specified (see Section 6), we will impose the following additional restrictions:

Assumption 3. *For all* j, $\sum_i \pi_{ij} = 1$.

In this way we discard the supply and demand imbalances at a specific location as the cause of a disruption in the payments system or in the interbank market.

Because of the complexity of the transfers involved in an arbitrary matrix Π, we will illustrate our findings in two symptomatic cases:

– In the first one $t_{ij} = 1$ if $j = i+1$ and $t_{ij} = 0$ otherwise, with the notational convention that $N+1 \equiv 1$. To visualize this case it is convenient to think that the consumers are located around a circle as in Salop's (1979) model. All travelers from i go to location $i+1$, the clockwise adjacent location, where they must consume at $t = 2$. The payments structure implied by this travel pattern generates what we define as *credit chain interbank funding,* when the bank at location $i+1$ provides credit to the incoming depositors from location i.

– In the second travel pattern $t_{ij} = \frac{1}{N-1}$ with $i \neq j$. Each two banks swap $\frac{\lambda}{N-1}$ customers so that at time $t = 2$ at location j there are $\frac{\lambda}{N-1}$ travelers from each of the other $(N-1)$ locations. We will refer to this perfectly isotropic case as the *diversified lending* case.[12]

With *credit chain interbank funding,* credit flows in the direction opposite to travel. With *diversified lending* every bank gives credit lines uniformly to all other $N-1$ banks. In terms of payments mecha-

[12] The structure of the payment flows in the credit chain interbank funding and in the diversified lending is very similar to that studied in Allen and Gale (1998).

nisms, the interbank credit described above can be interpreted as a compensation scheme (net system) or a Real Time Gross System (RTGS) with multilateral credit lines.

Let us now introduce the players of the game, namely the N banks and their depositors. At $t = 0$ banks decide whether to extend each other credit lines. In the absence of credit lines, all travelers have to withdraw at $t = 1$, which reduces the quantity that each bank can invest: this is what we call the *autarkic situation.* On the other hand, in the general case with credit lines, the value of final consumption at $t = 2$ is determined by a non-cooperative game played by the banks' depositors. At $t = 1$ each depositor located at i and consuming at location j simultaneously and without coordination determines the fraction x_{ij} of his or her deposit to maintain in the bank. Accordingly, the percentage of investment remaining at location j where he or she must consume is

$$X_j \equiv \max\left[1 - \sum_k \pi_{jk}(1 - x_{jk})\frac{D_0}{\alpha}, 0\right], \tag{3}$$

Because of Assumption 1, the final consumption of depositors (i, j) results from a combination of a withdrawal at time $t = 1$ in bank i (i.e. $(1 - x_{ij})D_0$) plus a proportion x_{ij} of the value at $t = 2$ of a deposit D_j in bank j. To determine the possible equilibria of the depositors' game, we have to compare D_0 with the (endogenous) values of the deposits $D_1, \ldots, D_N$ in all the banks at $t = 2$. Now, to determine D_i, consider the balance sheet equation for bank i at time $t = 2$:

$$X_i R_i + \sum_j \pi_{ji} x_{ji} D_j = \left(\sum_j \pi_{ji} x_{ji} + \sum_j \pi_{ij} x_{ij}\right) D_i \tag{4}$$

where the LHS (RHS) represents the assets (liabilities) of bank i, $X_i R_i$ is the return on its investment, $\sum_j \pi_{ji} x_{ji} D_j$ are the credits of bank i due from other banks, $\left(\sum_j \pi_{ji} x_{ji}\right) D_i$ are its debts with other banks, and $\sum_j \pi_{ij} x_{ij} D_i$ are its deposits. Notice that Assumption 2 implies that the above equation does not apply when bank i is closed at $t = 1$. In this case $X_i = D_i = 0$.

The optimal behavior of each depositor (i, j) is $x_{ij} = 1 \Leftrightarrow D_j \geq D_0, \forall i, j$. Since it depends only on j, we denote by x^j, the common value of the x_{ij} where $x^j = 1$ if $D_j > D_0$ and $x^j = 0$ otherwise. This allows a simplification of (4):

$$X_i R_i + \left(\sum_j \pi_{ji} D_j\right) x^i = \left[\left(\sum_j \pi_{ji}\right) x^i + \sum_j \pi_{ij} x^j\right] D_i. \tag{5}$$

We establish the following notation: $\mathbf{D} = (D_1, \ldots, D_N)'$, $\mathbf{R} = (R_1, \ldots, R_N)'$, and Π' is the transpose of $\Pi = (\pi_{ij})_{i,j}$. For a given strategy vector $(x_{ij})_{i,j}$ one can compute the assets in place at bank i (X_i) and the return on a deposit at bank i (D_i). Then we check whether the strategies are optimal:

$$x^*_{ij} = \begin{cases} 1 \text{ if } D_j > D_0 \\ 0 \text{ if } D_j < D_0 \end{cases} . \tag{6}$$

Any fixed point of this algorithm $(\text{i.e., } x^*_{ij} \equiv x_{ij})$ is an equilibrium of our game.

When the mechanism of interbank credit functions smoothly, $x_{ij} \equiv 1$ for all (i, j) and depositors' welfare is greater than in the autarkic situation. This is because interbank credit lines allow each bank to keep a lower amount of liquid reserves and to invest more. However, the system is also more fragile. As we show in the next Sections, the non cooperative game played by depositors has other equilibria than $x_{ij} \equiv 1$.

3 PURE COORDINATION PROBLEMS

We first analyze the equilibria of the game when all deposits are invested at $t = 1$, investment returns are certain, and all banks are solvent so that the only issue is the coordination among depositors. Disregarding the mixed strategy equilibria where depositors are indifferent between withdrawing their deposits and transferring them to the recipient banks, we obtain our first result:

Proposition 3.1 We assume $R_i > D_0$ for all i (which implies that all banks are solvent). There are at least two pure strategy equilibria: (i) the inefficient bank run allocation where $\mathbf{x}^* = \mathbf{0}$ (Speculative Gridlock Equilibrium) and (ii) the efficient allocation where $\mathbf{x}^* = \mathbf{1}$ (Credit Line Equilibrium).

Proof. See the Appendix.

Several comments are in order. In the Credit Line Equilibrium there is no liquidation while in the Speculative Gridlock Equilibrium all the banks' assets are liquidated. Since liquidation is costly and all banks are solvent, the Credit Line Equilibrium dominates the Speculative Gridlock Equilibrium as well as any other equilibrium where some liquidation takes place. The Speculative Gridlock Equilibrium arises as a result of a coordination failure like in D-D. If depositors rationally anticipated at $t = 0$ a Speculative Gridlock Equilibrium, they would prefer the autarkic situation.

In the Credit Line Equilibrium with diversified lending, bank *i* extends credit lines to all the other banks and receives credit lines from them. In equilibrium the debt arising from bank *i*'s depositors at $t = 2$ using bank *i*'s credit lines with the other banks, is repaid at $t = 2$ by bank *i* serving the depositors from the other banks. It is precisely because the behavior of one bank's depositors is affected by the expectation of what the depositors going to the same location will do, that this equilibrium is vulnerable to a coordination failure. If the depositors in a sufficiently large number of banks believe that they will be denied consumption at the location where they have to consume, it is optimal for them to liquidate their investment, which makes it optimal for the depositors in all other banks to do the same. The Speculative Gridlock Equilibrium is related to the notion of Domino Effect that may arise in payments systems as a result of the settlement failure of some participant. Still, it may occur here even if all banks are solvent. Notice, that banks do not play any strategic role: only depositors play strategically.

From the efficiency viewpoint, when all the banks are solvent the Credit Line Equilibrium dominates autarky which in turn dominates the Speculative Gridlock Equilibrium.[13] Hence there is a trade-off between a risky interbank market based on interbank credit and a safe payment mechanism which foregoes investment opportunities.[14]

Both the Gridlock and the Credit Line Equilibria involve the use of credit lines. In both equilibria banks extend and honor credit lines up to the amount of their $t = 2$ resources. In the Speculative Gridlock Equilibrium it is not the banks that do not honor the credit lines, rather are the depositors that, by forcing the liquidation of the investment, reduce the amount of resources available at $t = 2$.

[13] When α=1 the last two are equivalent. The cost of the Gridlock Equilibrium is proportional to 1-α. Notice that autarky is equivalent to a payment system with fully collateralized credit lines like TARGET (Trans-European Automated Real-Time Gross Settlement Express Transfer), the payment system designed to handle transactions in the Euro area.

[14] For an analysis of this trade off in a related setting see Freixas and Parigi (1998). However, even a Real Time Gross System like TARGET is not immune to a systemic crisis. As Garber (1998) points out if there is a risk that a currency will leave the Euro currency area, the very infrastructure of TARGET where National Central Banks of the participating countries extend to each other unlimited daily credit, provides the perfect mechanism to mount speculative attacks on the system.

There is a clear parallel between these two equilibria in our economy with N locations and the equilibria in a one-location D-D model. These results are also related to the papers by Bhattacharya and Gale (1987) and Bhattacharya and Fulghieri (1994) that consider N-location D-D economies without geographic risks.

The Credit Line Equilibrium can be implemented in several ways: through a Compensation System where credits are netted, by a RTGS (Real Time Gross Settlement) system with multilateral or bilateral credit lines, through lending by the Central Bank and through Deposit Insurance.

In this basic version of the model, in the event of a gridlock, every bank is *solvent* although *illiquid.* Thus no difficulty in distinguishing between insolvent and illiquid banks arises for the Central Bank.[15] The Central Bank has a simple coordinating role as a LOLR in guaranteeing private-sector credit lines or in providing fiat money, both backed by the authority of the Treasury to tax the return on the investment.[16]

Similarly, by guaranteeing the value of deposits at the consumption locations, Deposit Insurance eliminates any incentive for the depositors to protect themselves by liquidating the investment, thus making it optimal for banks to extend credit to each other.

Like Deposit Insurance which is never used in equilibrium in the D-D model, the coordination role of the Central Bank costs no resources (excluding moral hazard issues), since in equilibrium it will not be necessary for the Central Bank to intervene. However, in a richer model credit line guarantees and Deposit Insurance would not have the same effect. In fact, unlike credit lines guarantees, Deposit Insurance penalizes the managers of distressed banks, and might offer better incentives to managers to monitor each other.

[15] For an analysis of this issue see the companion paper by Freixas, Parigi, and Rochet (1998).

[16] For example in the Canadian electronic system for the clearing and settlement of large value payments the Central Bank guarantees intraday credit lines (Freedman and Goodlet 1998).

4 RESILIENCY AND MARKET DISCIPLINE IN THE INTERBANK SYSTEM

In the next two Sections we tackle the issue of the impact of the insolvency of one bank on the rest of the system. In this Section we investigate under which conditions the losses of one bank can be absorbed by the other banks without provoking withdrawals by depositors, (this is what we call resiliency) and what are the implications in terms of market discipline. In the next Section we consider the issue of contagion. That is we investigate whether the closure of an insolvent bank generates a chain reaction causing the liquidation of solvent banks.

In order to model the possibility of insolvency in a simple way, we make the extreme assumption that the return R_i on the investment at location i can be either $R \geq D_0$, or 0. If $R = 0$, bank i is insolvent, in which case it is efficient to liquidate it, absent contagion issues. For the remainder of this paper we assume that the probability of $R = 0$ is sufficiently low that it is optimal for the banks to invest all deposits at $t = 0$.[17] Returns are publicly observable at $t = 1$ but verifiable only at $t = 2$ so that no contract can be made contingent on these returns. Notice that by assumption 1 the public information that bank 1 is insolvent cannot be used the other banks to distinguish and discriminate the depositors of the insolvent bank. The efficient allocation of resources requires that banks be liquidated if and only if they are insolvent:

$$X_i = \begin{cases} 0 \text{ if } R_i = 0 \\ 1 \text{ if } R_i = R \end{cases} . \tag{7}$$

Whether this efficient closure rule is a Nash Equilibrium of the non-cooperative game between depositors, will depend on the structure of the interbank payment system. To illustrate this, we focus on the case in which one bank (say, bank 1) is insolvent, and we investigate under which conditions $x = (1, \ldots, 1)$ is still an equilibrium, i.e. under which conditions $D_i \geq D_0$ for all i. When $x = (1, \ldots, 1)$ and $R_1 = 0$ the

[17] For a large probability of failure, it is optimal to use the storage technology only.

balance sheet equations (5) give

$$\mathbf{D} = (2I - \Pi')^{-1} \begin{pmatrix} 0 \\ R \\ \dots \\ R \end{pmatrix} = R\,(2I - \Pi')^{-1} \begin{pmatrix} 0 \\ 1 \\ \dots \\ 1 \end{pmatrix}. \tag{8}$$

From (8) we define by γ the minimum of the components of the vector

$$(2I - \Pi')^{-1} \begin{pmatrix} 0 \\ 1 \\ \dots \\ 1 \end{pmatrix}. \tag{9}$$

We establish the following proposition:

Proposition 4.1 (Resiliency and Market discipline) When $R_1 = 0$, necessary condition for $x = (1, \dots, 1)$ to be an equilibrium is that the smallest value of time $t = 2$ deposits $(R\gamma)$, which depends on the structure of interbank payment flows, exceeds D_0.

Proof. From (8) and the definition of γ we see that $D_i \geq D_0$ for all i if and only if $\gamma \geq \frac{D_0}{R}$. □

Several comments are in order. Proposition 4.1 highlights an important aspect of the tension between efficiency and stability of the interbank system. On the one hand it establishes the conditions under which the system can absorb the losses of one bank without any deposit withdrawal. Resiliency, however, entails the cost of forbearance of the insolvent bank. On the other hand it establishes the conditions under which $x = (1, \dots, 1)$ is no longer an equilibrium. If a bank is known at $t = 1$ to be insolvent, depositors may withdraw and withdrawals may not be confined to the insolvent bank, hence market discipline entails the cost of possibly excessive liquidation. We interpret γ as a measure of the exposure of the interbank system as a whole to market discipline when one bank is insolvent.[18]

We now study how γ varies with λ (the proportion of travelers) and N (the number of locations) in the two cases of credit chain and diversified lending.

[18] As a benchmark consider again the limit of the D-D optimal contract when the proportion of early diers tends to zero. If we compute D_0 when $u(c) = c^{1-a}/(1-a)$ (CRRA utility function), from (1) we have $\frac{D_0}{R} = R^{-a}$, which is decreasing in R. Therefore more profitable assets decrease the exposure of the bank to market discipline.

Proposition 4.2 Both in the credit chain case and in the diversified lending case, γ increases with λ and N; i.e. when the proportion of travelers increases or the number of banks increases, the system becomes less exposed to market discipline.

Proof. See the Appendix.

When the number of banks increases, the insolvency of one bank has a lower impact on the value of the deposits in the other banks. Similarly an increase in the fraction of travelers spreads on the other banks a larger fraction of the loss due to the insolvency of one bank. This seems quite intuitive for the diversified lending case, since the banks hold more diversified portfolios of loans. The novelty is that this result holds true also for the credit chain case where banks have the possibility to pass part of their losses to other banks through the interbank market.

We now compare the two systems for given values of λ and N. We then compare the exposure to market discipline of the credit chain and the diversified lending structures.

Proposition 4.3 In case of the insolvency of one bank, the system is more exposed to market discipline under diversified lending than under credit chains; i.e. $\gamma^{CRE} > \gamma^{DIV}$.

Proof. See the Appendix.

Proposition 4.3 may appear counterintutive since diversification is usually associated with the ability to spread losses. The result depends on the proportion of the losses on its own portfolio that the insolvent bank is able to transfer to other banks through the payments system. In a diversified lending system there is more diversification so that solvent banks exchange a larger fraction of their claims. As a consequence in a diversified lending system the insolvent bank is able to pass over to the solvent banks a smaller fraction of its losses.

The case with three banks $(N = 3)$ and everybody travels $(\lambda = 1)$ provides a good illustration. In a diversified lending system the balance sheet equations (5) become:

$$D_i = \frac{1}{2}\left[R_i + \frac{1}{2}(D_{i-1} + D_{i+1})\right] \qquad i = 1, 2, 3. \tag{10}$$

This means that if bank 1 is insolvent (i.e. $R_1 = 0)$, depositors at banks 2 and 3 obtain an equal share of total surplus, while bank 1 depositors receive 50% less. After easy computations, we find that bank 1 depositors receive $\frac{2}{5}R$, or equivalently bank 1 is able to pass $\frac{3}{5}$ of its losses to the solvent banks whose depositors end up receiving $\frac{4}{5}R$.

Consider now the case of credit chains. Still assuming $\lambda = 1$, the balance sheet equations give:

$$D_i = \frac{1}{2}[R_i + D_{i+1}] \qquad i = 1, 2, 3. \tag{11}$$

We can compute the losses experienced by each bank (with respect to the promised returns R) and it is a simple exercise to check that the only solution is:

$$D_1 = \frac{3}{7}R; D_3 = \frac{5}{7}R; D_2 = \frac{6}{7}R. \tag{12}$$

Therefore, bank 1 is able to pass on a higher share of its losses than in the diversified lending case, which explains the lower exposure of the interbank system to market discipline in the credit chain system.

The results of this Section highlight another side of interbank markets in addition to their role in redistributing liquidity efficiently studied by Bhattacharya and Gale (1987). Interbank connections enhance the ”resiliency” of the system to withstand the insolvency of a particular bank. However, this network of cross liabilities may loosen market discipline and allow an insolvent bank to continue operating through the implicit subsidy generated by the interbank credit lines. This loosening of market discipline is the rationale for a more active role for monitoring and supervision with the regulatory agency having the right to close down a bank in spite of the absence of any liquidity crisis at that bank.

The effect of a Central Bank’s guarantee on interbank credit lines would be that $x = (1, \ldots, 1)$ is always an equilibrium, even if one bank is insolvent. The stability of the banking system would be preserved at the cost of forbearance of inefficient banks.

5 CLOSURE-TRIGGERED CONTAGION RISK

5.1 Efficiency vs. Contagion Risk

We now turn to the other side of the relationship between efficiency and stability of the banking system, and investigate under which conditions the closure at time $t = 1$ of an insolvent bank does not trigger the liquidation of solvent banks in a contagion fashion. Suppose that bank k is closed at $t = 1$. Assumption 2 implies that $X_k = 0$ and $D_k = 0$. Closing bank k at $t = 1$ has two consequences. First, we have an *unwinding* of the positions of bank k since $\pi_{ki}D_k$ assets and $\pi_{ki}D_i$ liabilities disappear from the balance sheet of bank k. In a richer setting this is equivalent to a situation in which the other banks have reneged on their credit lines toward bank k, possibly as a result of the arrival on negative signals on its return. Second, a proportion π_{ik} of travelers going to location k will be *forced to withdraw early* the amount $\pi_{ik}D_0$ and bank i will have to liquidate the amount $\pi_{ik}\frac{D_0}{\alpha}$. If $\pi_{ik}\frac{D_0}{\alpha}$ is sufficiently large bank i is closed at $t = 1$, otherwise the cost at $t = 2$, of the early liquidation is $\pi_{ik}\left(\frac{D_0}{\alpha}R - D_i\right)$.

Notice that if $\pi_{ik}\frac{D_0}{\alpha} \geq 1$ then $X_i = 0$, i.e. bank i is liquidated simply because there are too many depositors going from location i to location k, the bank is closed at $t = 1$. The type of contagion that takes place here is of a purely mechanical nature stemming simply from the direct effect of inefficient liquidation. Since this case is straightforward let us instead concentrate on the other case, namely $\pi_{ik}\frac{D_0}{\alpha} < 1$. Because of unwinding and forced early withdrawal, the full general case is more complex. Since $x^k = 0$, we have to suppress all that concerns bank k from the equations (5). We obtain:

$$X_{i\{k\}}R_i + \sum_{j\neq k}\pi_{ji}D_j x^i = \left(\sum_{j\neq k}\pi_{ij}x^j + \sum_{j\neq k}\pi_{ji}x^i\right)D_i, \tag{13}$$

where

$$X_{i\{k\}} = \max\left[1 - \pi_{ik}\frac{D_0}{\alpha} - \sum_{j\neq k}\pi_{ji}(1 - x^j)\frac{D_0}{\alpha}, 0\right]. \tag{14}$$

We now have to check whether $x_{ij} \equiv 1$ for all $i, j \neq k$, can correspond to an equilibrium. In this case, $X_{i\{k\}} = \max[1 - \pi_{ik}\frac{D_0}{\alpha}, 0]$ and system (13) becomes:

$$R_i = \left(\sum_{j \neq k} \frac{\pi_{ij} + \pi_{ji}}{X_{i\{k\}}} \right) D_i - \sum_{j \neq k} \frac{\pi_{ji}}{X_{i\{k\}}} D_j. \tag{15}$$

Since by assumption $R_i \equiv R$ for all $i \neq k$, (15) becomes

$$(1 - \pi_{ik} \frac{D_0}{\alpha})R + \sum_{j \neq k} \pi_{ji} D_j = (2 - \pi_{ik} - \pi_{ki}) D_i \tag{16}$$

This allows us to establish a result analogous to Proposition 4.1.

Proposition 5.1 (Contagion Risk) There is a critical value of the smallest time $t = 2$ deposits below which the closure of a bank causes the liquidation of at least another bank. This critical value is lower in the credit chain case than in the diversified lending case. The diversified lending structure is always stable when the number N of banks is large enough whereas N has no impact on the stability of the credit chain structure.

Proof. It follows the same structure of the proof of Proposition 4.1. Denoting by M_k the inverse of the matrix defined by system (16), stability is equivalent to:

$$\begin{pmatrix} D_1 \\ \dots \\ D_N \end{pmatrix} = RM_k \begin{pmatrix} 1 \\ \dots \\ 1 \end{pmatrix} > D_0 \begin{pmatrix} 1 \\ \dots \\ 1 \end{pmatrix}. \tag{17}$$

One can see that all the elements of M_k are non negative[19], thus stability obtains iff $\frac{D_0}{R} \leq \psi_k$, where ψ_k denotes the minimum of the components $M_k \begin{pmatrix} 1 \\ \dots \\ 1 \end{pmatrix}$. The computation of ψ_k is cumbersome in the general case but easy in our benchmark examples (where, because of symmetry, k does not play any role). One finds:

[19] The fact that the matrix M_k has non negative elements follows from a property of diagonal dominant matrices (See e.g. Takayama 1985 p.385).

$$\Psi_{cre} = 1 - \lambda\left(\frac{D_0}{\alpha} - 1\right); \Psi_{div} = 1 - \lambda\left(\frac{\frac{D_0}{\alpha} - 1}{N - 1 - \lambda}\right) \tag{18}$$

in the credit chain example, and in the diversified lending case, respectively. It is immediate from these formulas that $\Psi_{cre} < \Psi_{div}$ (for $N \geq 2$) and that Ψ_{div} tends to 1 when N tends to infinity while Ψ_{cre} is independent of N. □

5.2 **Comparison with Allen and Gale (1998)**

It is useful to compare our results with those of Allen and Gale (1998). Proposition 4.1 establishes that systemic crises may arise for fundamental reasons, like in Allen and Gale. However, the focus of the two papers is different. Allen and Gale are concerned with the stability of the system with respect to liquidity shocks arising from the random number of consumers that need liquidity early in the absence of aggregate uncertainty. They show that the system is less stable when the interbank market is incomplete (in the sense that banks are allowed to cross hold deposits only in a credit chain fashion) than when the interbank market is complete (in the sense that banks are allowed to cross hold deposits in a diversified lending fashion).

In our paper interbank links arise, instead, from consumers geographic uncertainty and the focus is on the implications of the insolvency of one bank in terms of market discipline and stability of the system. In particular in Proposition 4.3 we show how the structure of interbank links allows to spread over other banks the losses of one bank. We show that a diversified lending system is more exposed to market discipline (i.e. less resilient) than a credit chain system because in the latter the insolvent bank is able to transfer a larger fraction of its losses to other banks thus reducing the incentives for its own depositors to withdraw. In Proposition 5.1 we are concerned with the stability of the system with respect to contagion risk triggered by the efficient liquidation at time $t = 1$ of the insolvent bank.

6 TOO-BIG-TO-FAIL AND MONEY CENTER BANKS

Regulators have often adopted a too-big-to-fail approach (TBTF) in dealing with financially distressed money center banks and large financial institutions.[20] One of the reasons is the fear of the repercussions that the liquidation of a money center bank might have on the corresponding banks that channel payments through it. Our general formulation of the payments needs where the flow of depositors going to the various locations is asymmetric offers a simple way to model this case and to capture some of the features of the TBTF policy. We interpret the TBTF policy as designed to rescue banks which occupy key positions in the interbank network, rather than banks simply with large size.[21]

Consider for example the case where there are three locations $(N = 3)$. Locations 2 and 3 are peripheral locations and location 1 is a money center location. All the travelers of locations 2 and 3 must consume at location 1, and one half of the travelers of location 1 consume at location 2 and the other half at location 3. That is $t_{12} = t_{13} = \frac{1}{2}$ and $t_{21} = t_{31} = 1$, $t_{23} = t_{32} = 0$.[22] This implies that

$$X_1 = \max\left\{1 - \frac{D_0}{\alpha}\left[1 - (1-\lambda)x^1 - \lambda\left(\frac{x^2 + x^3}{2}\right); 0\right]\right\}; \tag{19}$$

and

$$\begin{aligned} X_2 = &\quad \max\left\{1 - \tfrac{D_0}{\alpha}\left[1 - (1-\lambda)x^2 - \lambda x^1\right]; 0\right\} \\ X_3 = &\quad \max\left\{1 - \tfrac{D_0}{\alpha}\left[1 - (1-\lambda)x^3 - \lambda x^1\right]; 0\right\}. \end{aligned} \tag{20}$$

Suppose now that one of these banks (and only one) is insolvent (this is known at $t = 1$). The next proposition illustrates how the closure of a bank with a key position in the interbank market may trigger a systemic crisis.

Proposition 6.1 (i) If $\lambda > \alpha(\frac{1}{D_0} - \frac{1}{R})$ the liquidation of bank 1 triggers the liquidation of all other banks (Too-big-to-fail); (ii) If $\lambda < \frac{2\alpha}{D_0}$, liquidation of banks 2 or 3 does not trigger the liquidation of any of the other two banks.

[20] See for example the intervention of the monetary authorities in the Continental Illinois debacle in 1984 and, to some extent, in arranging the private-sector rescue of Long Term Capital Management.

[21] The Barings' failure of 1996 is an example of the crisis of a large financial institution that did not create systemic risk.

[22] Notice that we now abandon Assumption 3 (the symmetry assumption).

Proof. To prove (i) notice that if bank 1 is closed then $X_1 = 0$, and $x^1 = 0$. Then $D_2 = X_2 R = (1 - \frac{D_0}{\alpha}\lambda)R$. Thus $x^2 = 0$ if $(1 - \frac{D_0}{\alpha}\lambda)R < D_0 \Leftrightarrow \lambda > \alpha(\frac{1}{D_0} - \frac{1}{R})$. To prove (ii) notice that if bank 2 is closed then $x^2 = 0$. If $(1,0,1)$ is an equilibrium the balance sheet equations become, when $\frac{D_0\lambda}{\alpha} < 2$:

$$
\begin{aligned}
D_1\left(1 - \frac{\lambda}{2}\frac{D_0}{\alpha} + \lambda\right) &= \left(1 - \frac{D_0}{\alpha}\frac{\lambda}{2}\right)R_1 + \lambda D_3 \qquad (21)\\
D_3\left(1 + \frac{\lambda}{2}\right) &= R_3 + \frac{\lambda}{2}D_1.
\end{aligned}
$$

If $R_3 = R_1 = R$ this yields $D_3 = D_1 = R$. This implies that $x = (1,0,1)$ is an equilibrium whenever $\frac{D_0\lambda}{\alpha} < 2$. □

Our last result concerns the optimal attitude of the Central Bank when the money center bank becomes insolvent $(R_1 = 0)$. When $\frac{D_0}{R}$ is low, no intervention is needed. When $\frac{D_0}{R}$ is large, the Central Bank has to inject liquidity. More precisely we have:

Proposition 6.2 When $R_1 = 0, x = (1,1,1)$ is an equilibrium if $\frac{D_0}{R}$ is sufficiently low (no Central Bank intervention is needed). In the other case, the cost of bailout increases with $\frac{D_0}{R}$.

Proof. When $R_1 = 0, x = (1,1,1)$ can be an equilibrium if $\mathbf{D} > D_0 \begin{pmatrix} 1 \\ \dots \\ 1 \end{pmatrix}$. When $x = (1,1,1)$, the balance sheet equations (5) become

$$
\begin{aligned}
R_1 + (D_2 + D_3) &= 3D_1 \qquad (22)\\
R_2 + \frac{1}{2}D_1 &= \frac{3}{2}D_2, R_3 + \frac{1}{2} = \frac{3}{2}D_3. \qquad (23)
\end{aligned}
$$

Solving (22) and (6.2) when $R_1 = 0, R_2 = R_3 = R$ yields $D_1 = \frac{4}{7}R, D_2 = D_3 = \frac{6}{7}R$, which is an equilibrium iff $\frac{D_0}{R} < \frac{4}{7}$. The cost of bailout is 0 iff $\frac{D_0}{R} < \frac{4}{7}$, it is $D_0 - \frac{4}{7}R$ iff $\frac{4}{7} < \frac{D_0}{R} < \frac{6}{7}$. When $\frac{D_0}{R} > \frac{6}{7}$, the Central Bank also has to inject liquidity in the solvent banks. The total cost to the Central Bank becomes $3D_0 - \frac{16}{7}R$. □

7 DISCUSSIONS AND CONCLUSIONS

We have constructed a model of the banking system where liquidity needs arise from consumers' uncertainty about where they need to consume. Our basic insight is that the interbank market allows to minimize the amount of resources held in low-return liquid assets. However, interbank links expose the system to the possibility that a number of inefficient outcomes arise: the excessive liquidation of productive investment as a result of coordination failures among depositors; the reduced incentive to liquidate insolvent banks because of the implicit subsidies offered by the payments networks; the inefficient liquidation of solvent banks because of the contagion effect stemming from one insolvent bank.

7.1 **Policy implications**

We use this rich set-up to derive a number of policy implications (summarized in Table 1) with respect to the interventions of the Central Bank.

First, the interbank market may not yield the efficient allocation of resources because of possible coordination failures that may generate a ”gridlock” equilibrium. The Central Bank has thus a natural coordination role to play which consists of implicitly guaranteeing the access to liquidity of individual banks. If the banking system as a whole is solvent the costs of this intervention is negligible and its distortionary effects may stem only from moral hazard issues (Proposition 3.1).

Table 1. Summary of Central Bank Interventions

Problem	*Type of Central Bank Intervention*	*Costs*	*Result*
Speculative Gridlock	Coordinating role of Central Bank • Guarantee credit lines • Deposit Insurance	Never used in equilibrium; no cost, apart from moral hazard	Proposition 3.1
Insolvency in a resilient interbank market	Ex ante monitoring and supervision	Imperfect monitoring leads to forebearance and moral hazard	Proposition 4.1
Insolvency leading to contagion	Orderly closure of insolvent bank and arrangement of credit lines to by-pass it	No cost, apart from moral hazard and money center banks; in case of money center banks it may be too costly or even impossible to organize orderly closure	Proposition 5.1; Proposition 6.1
	Bail out	Transfer of taxpayer money	Proposition 6.2

Second, if one bank is insolvent, the Central Bank faces a much more complex trade-off between efficiency and stability. Market forces will not necessarily force the closure of insolvent banks. Indeed the resiliency of the interbank market allows to cope with liquidity shocks of individual banks by providing implicit insurance, which weakens market discipline (Proposition 4.1). Therefore the Central bank has the responsibility to provide ex ante monitoring of individual banks. However, the closure of insolvent banks may cause systemic repercussions (Proposition 5.1) which is the responsibility of the Central Bank to handle. In this case two courses of actions are available: orderly closure or bailout of insolvent banks. Given the interbank links, the closure of an insolvent bank must be accompanied by the provision of Central Bank liquidity to the counterparts of the closed bank.[23] This is what we called orderly closure. Assuming that this is possible, theoretically it entails no costs apart from moral hazard. However, the orderly closure might simply not be feasible for money center banks (Proposition 6.1) in which case

[23] For instance, in the credit chain case, if bank k is closed the Central Bank can borrow from bank $k-1$ and lend to bank $k+1$, thus allowing the interbank arrangements to function smoothly.

the Central Bank has no choice but to bailout the insolvent institution, with the obvious moral hazard implications of the TBTF policy.

Our model can be extended in various directions some of which are discussed below.

7.2 Imperfect Information on Banks' Returns

In reality, both the Central Bank and the depositors have only imperfect signals on the solvency of commercial banks (although the Central Bank' signals are hopefully more precise). Therefore, the Central Bank will have to act knowing that with some probability it will be lending to (guaranteeing the credit lines of) insolvent institutions and with some probability it will be denying credit to solvent institutions. Also, depositors may run on all the banks which have generated a bad signal.

The consequences are different depending on the structure of the interbank market. In the credit chain case, the Central Bank will have to intervene to provide credit with a higher probability than in the diversified lending case. Therefore in the credit chain case the Central Bank has a higher probability of ending up financing insolvent banks. Ex ante, therefore, the Central Bank intervention is much more expensive in the credit chain case, so that in this case a fully collateralized payments system may be preferred.

7.3 Payments among different countries

Systemic risk is often related to the spreading of financial crisis from one country to another. Our basic model can be extended to consider various countries instead of locations within the same country. When depositors belong to different countries, travel patterns that generate a consumption need in another location have the natural interpretation of demand of goods of other countries, i.e. import demand.

Goods of the other country can be purchased through currency (like in autarchy in the basic model) or through a credit line system whereby the imports of a country are financed by its exports.

Our results extend to the model with different countries but the role of the monetary authority is somewhat different. While in our set-up the lending ability of the domestic monetary authority was backed by its taxation power, the lending ability of an international financial organization is ultimately backed by its capital. Hence the resources at its disposal are limited and in case of aggregate uncertainty its ability to guarantee banks' credit lines is limited.[24]

[24] See the role of the I.M.F. in the 1997 Asian crises and the 1998 Russian crisis.

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APPENDIX

Notation. Define

$$M(\lambda) \equiv \left[2I - \Pi'\right]^{-1} = \left[(1+\lambda)I - \lambda T'\right]^{-1} = \tag{24}$$

$$= \frac{1}{1+\lambda}\left[I - \frac{\lambda}{1+\lambda}T'\right]^{-1}$$

where I is the identity matrix. We first need a technical lemma:

Lemma 7.1 All the elements of $M(\lambda)$ are non negative: $m_{ij}(\lambda) \geq 0$ for all i, j. Moreover for all i, $\sum_j m_{ij}(\lambda) = 1$. As a consequence, if $R_i > D_0$ for all i then

$$M(\lambda)\mathbf{R} > D_0 \begin{pmatrix} 1 \\ \ldots \\ 1 \end{pmatrix}. \tag{25}$$

Proof. $M(\lambda) = (2I - \Pi')^{-1}$. Since Π' is a Markov matrix (because of assumption 3), all its eigen values are in the unit disk and $M(\lambda)$ can be developed into a power series:

$$M(\lambda) = \frac{1}{2}\left(I - \frac{\Pi'}{2}\right)^{-1} = \sum_{k=0}^{+\infty} \frac{\Pi'^k}{2^{k+1}}. \tag{26}$$

This implies that $M(\lambda)$ has positive elements. Moreover $\begin{pmatrix} 1 \\ \ldots \\ 1 \end{pmatrix}$ being an eigen vector of Π' (for the eigen value 1), it is also an eigen vector for $M(\lambda)$. □

Proof of Proposition 3.1.

(i) Because of assumption 2, $D_i = 0$ when $x_{ij} = 0$ for all j. Therefore $x^*_{ij} \equiv 0$ is always an equilibrium.

(ii) $x^j = 1 \Rightarrow X_j = 1$. Using the assumption that $\sum_j \pi_{ji} = 1$ equation (5) becomes

$$2\mathbf{D} = \mathbf{R} + \Pi'\mathbf{D}. \tag{27}$$

For $x^j = 1$ to be an equilibrium for all j, it must be

$$\mathbf{D} = \left[2I - \Pi'\right]^{-1} \mathbf{R} = M(\lambda)\,\mathbf{R} \geq D_0 \begin{pmatrix} 1 \\ \dots \\ 1 \end{pmatrix}. \tag{28}$$

This is an immediate consequence of the above lemma, which implies that $x = (1, \dots, 1)$ is always an equilibrium when all banks are solvent. There are no other equilibria when $\alpha = D_0$. Indeed if $x^i = 0$ then equation (5) implies that $X_i = 0$ or $D_i = R_i$. But X_i cannot be zero (unless all x^j are also zero) and $D_i = R_i > D_0$ contradicts the equilibrium condition. Notice, however, that when $\alpha < D_0$, X_i can be zero even if some of the x^j are positive, which implies that other equilibria may exist. □

Before establishing Proposition 4.2 we have to compute the expression of matrix $M(\lambda)$ in the two cases of credit chain and diversified lending.

Consider the credit chain case first, where the matrix T is given by:

$$T = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ & & \dots & & \\ & & & \dots & \\ 0 & \dots & & 0 & 1 \\ 1 & 0 & \dots & 0 & 0 \end{pmatrix} \tag{29}$$

Therefore $T'^N = I$, so that $T'^k = T'^{k+N} = T'^{k+2N} \dots$. Now

$$M(\lambda) = \left(\frac{1}{1+\lambda}\right) \sum_{k=0}^{\infty} \left(\theta T'\right)^k, \tag{30}$$

where $\frac{\lambda}{1+\lambda} \equiv \theta$. Let $\Theta \equiv \left\{1 + \theta + \theta^N + \theta^{2N} \dots\right\}$. Thus

$$M(\lambda) \equiv \frac{\Theta}{1+\lambda} \left[I + \theta T' + \left(\theta T'\right)^2 + \dots + \left(\theta T'\right)^{N-1}\right] = \frac{1-\theta}{1-\theta^N} A \tag{31}$$

where

$$A \equiv \left[I + \theta T' + \ldots + (\theta T')^{N-1}\right] = \begin{pmatrix} 1 & \theta^{N-1} & \ldots & \ldots & \theta^2 & \theta \\ \theta & 1 & \theta^{N-1} & \ldots & \ldots & \theta^2 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & 1 & \theta^{N-1} \\ \theta^{N-1} & \ldots & \ldots & \theta^2 & \theta & 1 \end{pmatrix} \tag{32}$$

Consider now the diversified lending case, where the matrix T is given by:

$$T = \frac{1}{N-1}\begin{pmatrix} 0 & 1 & \ldots & \ldots & 1 \\ 1 & 0 & 1 & \ldots & 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & \ldots & 1 & 0 & 1 \\ 1 & \ldots & \ldots & 1 & 0 \end{pmatrix}. \tag{33}$$

It follows that $T = T'$. Now

$$M(\lambda) = \frac{1}{1+\lambda}\left[I - \frac{\lambda}{1+\lambda}T'\right]^{-1} = (1-\theta)\sum_{k=0}^{\infty}(\theta T')^k. \tag{34}$$

Notice that

$$T'^2 = \frac{1}{N-1}I + \frac{N-2}{N-1}T';$$

$$T'^3 = \frac{1}{N-1}T' + \frac{N-2}{N-1}T'^2 = \frac{1}{N-1}T' + \frac{N-2}{N-1}\left[\frac{1}{N-1}I + \frac{N-2}{N-1}T'\right].$$

Finally,

$$T'^3 = \frac{N-2}{(N-1)^2}I + \left[1 - \frac{N-2}{(N-1)^2}\right]T'. \tag{35}$$

Recursively we obtain

$$T'^{k} = \beta_k I + (1-\beta_k)\, T' \tag{36}$$

where

$$\beta_k = \frac{1}{N}\left[1 - \left(\frac{-1}{N-1}\right)^{k \quad 1}\right]. \tag{37}$$

Therefore

$$M(\lambda) = (1-\theta)\sum_{k=0}^{\infty}\left(\theta T'\right)^{k} = (1-\theta)\sum_{k=0}^{\infty}\left[\theta^{k}\beta_k I + \theta^{k}(1-\beta_k)\, T'\right] \tag{38}$$

Proof of Proposition 4.2. If $\mathbf{R} = \begin{pmatrix} 0 \\ R \\ \ldots \\ R \end{pmatrix}$ the necessary condition for $x = (1, \ldots, 1)$ to be an equilibrium becomes

$$\mathbf{D} = M(\lambda)\,\mathbf{R} = M(\lambda)\begin{pmatrix} 0 \\ R \\ \ldots \\ R \end{pmatrix} \geq D_0. \tag{39}$$

In the credit chain case equation (32) implies that the first row of condition (39) becomes

$$\frac{1-\theta}{1-\theta^{N}}(\theta^{N \quad 1} + \ldots + \theta)R \geq D_0 \tag{40}$$

or

$$\frac{D_0}{R} \leq 1 - \frac{1}{1+\theta+\ldots+\theta^{N \quad 1}} \equiv \gamma_N^{CRE} \tag{41}$$

It is easy to see that γ_N^{CRE} increases in N and in θ (and therefore in γ). Notice that $\gamma_\infty^{CRE} = \theta$.

Under diversified lending, $M(\lambda)$ is given by (38). Checking the first row of (39) and dividing by R yields

$$\frac{D_1}{R} = (1-\theta)\sum_{k=1}^{\infty}\left[\theta^{k}(1-\beta_k)\frac{N-1}{N-1}\right] \equiv \gamma_N^{DIV} \geq \frac{D_0}{R}. \tag{42}$$

Using

$$\beta_k = \frac{1}{N}\left[1-\left(\frac{-1}{N-1}\right)^{k-1}\right]. \qquad (43)$$

equation (42) becomes

$$\gamma_N^{DIV} = (1-\theta)\sum_{k=1}^{\infty}\theta^k\left(1-\frac{1}{N}\left[1-\left(\frac{-1}{N-1}\right)^{k-1}\right]\right). \qquad (44)$$

or

$$N\gamma_N^{DIV} = (1-\theta)\left[(N-1)\sum_{k=1}^{\infty}\theta^k + \sum_{k=1}^{\infty}\theta^k\left(\frac{-1}{N-1}\right)^{k-1}\right]. \qquad (45)$$

Since

$$(1-\theta)\sum_{k=1}^{\infty}\theta^k = \frac{(1-\theta)\,\theta}{(1-\theta)} = \theta. \qquad (46)$$

and

$$\begin{aligned}(1-\theta)\sum_{k=1}^{\infty}\theta^k\left(\frac{-1}{N-1}\right)^{k-1} &= \theta(1-\theta)\sum_{k=0}^{\infty}\theta^k\left(\frac{-1}{N-1}\right)^{k} \\ &= \frac{\theta(1-\theta)}{1+\frac{\theta}{N-1}} = \frac{(N-1)\,\theta\,(1-\theta)}{N-1+\theta}.\end{aligned} \qquad (47)$$

equation (45) becomes

$$N\gamma_N^{DIV} = (N-1)\,\theta + \frac{(N-1)\,\theta\,(1-\theta)}{N-1+\theta} = \frac{(N-1)\,\theta\,[N-1+\theta+1-\theta]}{N-1+\theta} \qquad (48)$$

from which

$$\gamma_N^{DIV} = \frac{(N-1)\theta}{N-1+\theta} = \frac{1}{\frac{1}{\theta}+\frac{1}{N-1}}. \qquad (49)$$

Recalling that $\theta = \frac{\lambda}{1+\lambda}$, we see that γ_N^{DIV} increases with λ and N, and that $\gamma_\infty^{DIV} = \theta$. ☐

Proof of Proposition 4.3. Comparing γ_N^{DIV} and γ_N^{CRE} we obtain

$$\frac{\gamma_N^{DIV}}{\theta} = \frac{(N-1)}{N-1+\theta} = \frac{1}{1+\frac{\theta}{N-1}}. \qquad (50)$$

and

$$\frac{\gamma_N^{CRE}}{\theta} = \frac{1-\theta^{N-1}}{1-\theta^{N}} = \frac{1+\theta+\theta^2+\theta^3 \ldots + \theta^{N-2}}{1+\theta+\theta^2+\theta^3 \ldots + \theta^{N-1}} = \frac{1}{1+\frac{\theta^{N-1}}{1+\theta+\theta^2+\theta^3 \ldots + \theta^{N-2}}}. \tag{51}$$

Since $\theta^{N-2} < \theta^{N-3} < \theta^{N-4} < \ldots$, then

$$\frac{\theta^{N-2}}{1+\theta+\theta^2+\theta^3 \ldots + \theta^{N-2}} < \frac{1}{N-1}. \tag{52}$$

Thus

$$\frac{\theta^{N-1}}{1+\theta+\theta^2+\theta^3 \ldots + \theta^{N-2}} < \frac{\theta}{N-1} \Rightarrow \frac{\gamma_N^{CRE}}{\theta} > \frac{\gamma_N^{DIV}}{\theta} \tag{53}$$

□