

# Learning and Complementarities in Speculative Attacks\*

Itay Goldstein, Emre Ozdenoren, and Kathy Yuan<sup>†</sup>

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## Abstract

We study a model where the aggregate trading of currency speculators reveals new information to the central bank and affects its policy decision. We show that the learning process gives rise to coordination motives among speculators leading to large currency attacks and introducing non-fundamental volatility into exchange rates and policy decisions. We show that the central bank can improve the ex-ante effectiveness of its policy by committing to put a lower weight ex-post on the information from the market, and that transparency may either increase or decrease the effectiveness of learning from the market, depending on how it is implemented.

*Journal of Economic Literature* Classification Codes: G14, G15, F31, C7.

*Keywords:* Currency attacks, financial markets, global games, strategic complementarities, heterogenous information, learning, coordination, feedback effect.

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<sup>†</sup>Goldstein is from the University of Pennsylvania; e-mail: [itayg@wharton.upenn.edu](mailto:itayg@wharton.upenn.edu). Ozdenoren is from the London Business School and the University of Michigan; e-mail: [eozenoren@london.edu](mailto:eozenoren@london.edu) and [emreo@umich.edu](mailto:emreo@umich.edu). Yuan is from the London School of Economics; e-mail: [K.Yuan@lse.ac.uk](mailto:K.Yuan@lse.ac.uk).

# 1 Introduction

Aggregating the heterogeneous beliefs possessed by many speculators, financial markets provide useful information about economic fundamentals. It is therefore not surprising that central banks, like other decision makers, pay close attention to the market, trying to extract new information that will guide their decision process. Indeed, Piazzesi (2005) provides evidence that monetary policy is affected by market data.<sup>1</sup>

Despite the wide belief that financial markets play an informational role, the theoretical implications of the informational feedback from market activities to policy decisions are still not well understood. In this paper, we shed light on this issue by embedding informational feedback into a model of currency attacks. In the model, the central bank learns from the speculative trading in currency markets about the viability of its currency regime, and uses the inferred information to guide its policy decisions. While learning from the market is usually perceived as a positive step helping to improve the decision process of the central bank, our theoretical analysis uncovers a surprising result. The very fact that the central bank learns from the market turns out to facilitate coordination motives among speculators leading to large currency attacks and to regime changes that are not driven by fundamentals. Such attacks correspond well to documented evidence that speculative attacks and transitions between exchange rate regimes are sometimes difficult to explain with fundamentals (see, for example, Eichengreen, Rose, and Wyplosz (1995)).

Turning to the details of our model, we analyze a situation where a central bank makes a decision whether to maintain a previously-announced fixed exchange rate regime. The central bank wants to maintain the regime only if the fundamentals of the economy are strong enough to support it. The central bank is only partially informed about the fundamentals of the economy. Speculators in the currency market also have pieces of information, which they use when deciding whether to speculate against the currency regime. By observing speculators' activities, the central bank gets an aggregate picture of the pieces of information held by speculators. Seeing a large attack against the currency, the central bank may come to believe that the fundamentals are bad and thus that it should abandon the regime.<sup>2</sup>

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<sup>1</sup>Similarly, in a different context, Luo (2005) and Chen, Goldstein, and Jiang (2007) show that financial-market prices affect managerial real-investment decisions.

<sup>2</sup>The regime change decision in the model can be interpreted more broadly to capture other central bank policy decisions such as intervention in a managed-float exchange rate environment.

Coordination motives arise in this framework because speculators know that a large speculative attack against the regime has the potential of convincing the central bank that the fundamentals are weak and that the regime should be abandoned. Hence, the expectation of a large attack increases the incentive of an individual speculator to speculate against the regime. Strategic complementarities manifest themselves then as a desire of speculators to put more weight on signals that are correlated with other speculators' information. Assuming that speculators have access to two types of signal, one that is conditionally uncorrelated across speculators and one that is conditionally correlated (e.g., due to market-wide rumors), they put too much weight on the latter. This implies that noise in the speculators' correlated signals gets to have a large impact on aggregate market outcomes and on the central bank's policy decision (given that the central bank cannot fully tease out the correlated noise component from market data). Empirically, this leads to non-fundamental volatility in speculative attacks and regime shifts, which is consistent with the results in the empirical currency-crises literature mentioned above.

Traditional models explain currency attacks as a run on the foreign reserves of the central bank (see Salant and Henderson (1978), Krugman (1979), and Flood and Garber (1984)). In Obstfeld (1996) and Morris and Shin (1998), the fact that the government has limited reserves generates strategic complementarities among speculators: the attack by some speculators creates direct pressure on the government to abandon the regime, increasing the incentive of others to attack as well. Strategic complementarities in our framework are different. They arise endogenously as a result of the learning by the central bank and are not imposed on the payoff functions via a reserve constraint. Hence, we distinguish the *informational complementarities* in our framework from the *direct complementarities* in the previous literature.

While both direct complementarities and informational complementarities are able to produce the non-fundamental volatility observed in the data, our model has an advantage in dealing with several aspects of the data. First, over the years many researchers and commentators have argued that the strong dependence on reserves, as postulated in traditional currency-attack models, is unrealistic (see, for example, Drazen (2000) and Krugman (2000)). Our model shows that coordination motives and non-fundamental volatility can arise even if the reserve constraint is not binding. Note that while our model assumes no reserve constraints, this is not a necessary assumption. In the presence of reserve constraints, the informational complementarities and direct complementarities will amplify each other,

generating even larger effects.<sup>3</sup> What our paper shows is that reserve constraints are not necessary for complementarities to arise, as this happens through the learning process.

Second, the traditional approach is suited to explain only speculative attacks against an over-appreciated currency, since reserves do not pose a problem when the currency is over-depreciated. Our approach, on the other hand, can be used to analyze attacks against an over-depreciated currency, such as the Chinese Yuan, which has come under several rounds of attacks in recent years, some of them leading to significant revaluations.

Third, the assumption in traditional models that the central bank is fully informed before a speculative attack erupts is inconsistent with the dynamics of real-world currency attacks. In many attacks, the central bank first defends the currency regime for a period of time, and then abandons it.<sup>4</sup> A fully informed central bank would not bear the costs of defense just to eventually abandon the regime. Our approach rationalizes this phenomenon by introducing learning by the central bank about the fundamentals of the regime. In parallel and independent work, Kurlat (2008) provides a model that addresses this concern by assuming that the central bank learns from the currency attack about the types of speculators in the market. Some of the implications of our model, including the emergence of informational complementarities, extend to the case considered in Kurlat's (2008) paper. However, the learning about fundamentals, featured in our paper, generates somewhat different empirical implications, as our empirical predictions center on the amount of information about fundamentals possessed by different parties (see below), and so empirical testing of our model should involve proxies of fundamental uncertainty.

The idea that the central bank learns from speculative activities in the currency market about fundamentals is rooted in empirical research demonstrating the importance of private and heterogenous information in currency trading (see Ito, Lyons, and Melvin (1998), Lyons (2001), Evans and Lyons (2002), and Evans and Lyons (2009)). To the extent that the central bank is not perfectly informed about the state of the economy, it is expected that it will try to infer some information from the market.<sup>5</sup> The information can be on micro

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<sup>3</sup>An interesting possibility is that the market aggregates information about the severity of the reserves constraint (i.e., future access to international markets, etc.) and that the central bank incorporates this information in its decision about the currency regime.

<sup>4</sup>For example take the Bank of Italy in the ERM crisis of 1992. The bank attempted to defend the lira by increasing the interest rate by 1.75 percentage points. Eventually, it gave in to speculative pressure, and the lira was withdrawn from the ERM. Other banks followed similar paths.

<sup>5</sup>The notion of heterogenous and private information is slowly being introduced into models of currency

variables like firms' leverage ratios and banks' financial strength, on which speculators are likely to have useful private information. Such variables have been linked to currency attacks in the third-generation models that came after the Asian crisis in the late 1990s.<sup>6</sup> Also, when it comes to traditional macro variables like the current-account deficit or the terms of trade, different market participants have different views and interpretations about the future evolution of these variables and its effect on the viability of the currency regime. We expect the central bank to be able to learn from the assimilation of these dispersed views that is performed in the market and to incorporate the information in its policy decision.<sup>7</sup> This idea of information aggregation in market outcomes goes back to Hayek (1945).

Our analysis shows that the informational complementarities in our paper generate different implications from the usual direct complementarities. For example, while a decrease in the transaction cost of attacking the currency increases the probability that speculators will participate in a speculative attack, it has no effect on the probability that the central bank will abandon the regime. This is because the central bank cares only about the informational content of the attack, and thus filters out the effect that transaction costs have on its size. This implies that while some speculative attacks will be defended, others that are even weaker (but more informative) will not be defended.

Based on this logic, variables that characterize the quality of information turn out to have the most substantial effect in our model. When the idiosyncratic sources of the speculators' information become more precise, the quality of the information inferred by the central bank from the trading outcome increases and the central bank becomes more likely to make a decision that is justified by fundamentals. On the other hand, the precision of the common element in the speculators' information sets may have the opposite effect, as an increase in the precision of this source of information may increase the ability of speculators to coordinate and mislead the central bank.<sup>8</sup> Given the importance of information variables in our model,

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markets. See, for example, Morris and Shin (1998), Bacchetta and van Wincoop (2006), Angeletos and Werning (2006), Hellwig, Mukherji, and Tsyvinski (2006), Angeletos, Hellwig, and Pavan (2007), and Broner (2008). None of these papers, however, considers learning by the central bank.

<sup>6</sup>For example see: Krugman (1999), Burnside, Eichenbaum, and Rebelo (2001), Chang and Velasco (2001), and Goldstein (2004). Reviews that discuss the shift to micro variables in the currency-attack literature include Pesenti and Tille (2000) and Burnside, Eichenbaum, and Rebelo (2008).

<sup>7</sup>Recall that we do not assume individual speculators to be more informed than the central bank. Rather, it is the aggregation of their information sets that may provide useful information.

<sup>8</sup>The decrease in efficiency following an increase in the precision of a public signal is reminiscent of the

empirical tests of it should use measures from the market microstructure literature that characterize the amount of information in the trading process.

An interesting observation that comes out of our model is that learning from the market by the central bank is self-defeating. This is because the learning process is the source of the complementarities that end up reducing the informativeness of the market outcome. Thus, learning from the market exposes the central bank to a time-inconsistency problem that creates a benefit to commitment. In particular, we show that the central bank can improve the *ex-ante* effectiveness of learning from the market by committing to put a lower weight on the information conveyed by the speculative attack than is ex-post efficient. While this generates more errors ex-post, it changes the *ex-ante* incentives of speculators in a way that makes them put a lower weight on their correlated information. This, in turn, increases the informativeness of their aggregate action.

Another policy measure that we analyze is transparency by the central bank.<sup>9</sup> We consider the case where speculators commonly observe a signal of the central bank's information. We show that this reduces policy effectiveness because it provides common information to the speculators about the action that the central bank is likely to take, and this enables the speculators to coordinate on their common information more effectively. Interestingly, we show that if speculators heterogeneously interpret the central bank's communication, transparency has a positive effect on policy effectiveness because giving speculators better heterogeneous information reduces their ability to coordinate and leads them to reveal more accurate information through the attack. Hence, for transparency to be beneficial, the central bank needs to send a message that is interpreted differently by different speculators. It would be interesting to explore how this could be applied in practice.

Relating the mechanism in our model to the broader literature, we note that informational externalities are common in models of financial markets. Usually in these models, strategic substitutes arise among speculators, as the information that motivates a speculator to trade gets reflected in the price and discourages other speculators from trading or from acquiring information (see, e.g., Grossman and Stiglitz (1980)). However, there are few models that result in Morris and Shin (2002) and Angeletos and Pavan (2007), which was obtained in a framework with direct complementarities.

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<sup>9</sup>There is a large literature on transparency. Some recent works include Morris and Shin (2002, 2005), Heinemann and Cornand (2004), Woodford (2005), Svensson (2005), Hellwig (2005), Angeletos and Pavan (2007), and Amador and Weill (2007).

feature strategic complementarities. For example, in Froot, Scharfstein, and Stein (1992), and Allen, Morris, and Shin (2006), strategic complementarities arise from the assumption that agents have short horizons. In Amador and Weill (2009) and Ganguli and Yang (2009), informative prices may make speculators' private forecast more precise, leading them to rely more on their private forecast. This then becomes self-fulfilling because it makes prices more informative, generating the strategic complementarities. The source of complementarities in our model is quite different than the literature. Strategic complementarities among currency speculators in our model arise solely due to the feedback effect that the information in their trades has on the policy decision of the policy maker.

Focusing on this feedback effect, our paper is related to a small, but growing, branch of models in financial economics that consider the feedback effect from trading in financial markets to corporate investments. Earlier contributions to this literature include Fishman and Hagerty (1992); Leland (1992); Khanna, Slezak, and Bradley (1994); Boot and Thakor (1997); Dow and Gorton (1997); Subrahmanyam and Titman (1999); and Fulghieri and Lukin (2001). Several recent papers in this literature are more closely related to the mechanism in our paper. Ozdenoren and Yuan (2008) show that the feedback effect from asset prices to the real value of a firm generates strategic complementarities. In their paper, however, the feedback effect is modeled exogenously and is not based on learning. Goldstein and Guembel (2008) do analyze learning by a decision maker, and show that this might lead to manipulation of the price by a single potentially informed trader. Hence, the manipulation equilibrium in their paper is not a result of strategic complementarities among heterogeneously informed traders.<sup>10</sup> Dow, Goldstein, and Guembel (2007) show that the feedback effect generates complementarities in the decision to produce information, but not in the trading decision.<sup>11</sup> Overall, the new insight in our paper – informational complementarities in trading due to the feedback effect – has not been explored in this literature, and thus with proper modeling can lead to a new contribution in the context of corporate finance.<sup>12</sup>

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<sup>10</sup>See also Khanna and Sonti (2004), where manipulation happens as a result of the feedback effect. In their paper, feedback is exogenous and not based on learning.

<sup>11</sup>Complementarities in the decision to produce information also arise due to other reasons in several other papers. For example see Hirshleifer, Subrahmanyam, and Titman (1994); Bru and Vives (2002); and Veldkamp (2006a and 2006b).

<sup>12</sup>Our paper can be linked to contexts that are even beyond financial markets. For example, a typical problem in political economics involves a policymaker trying to learn from lobbying groups (e.g., in Battaglini and Benabou (2003)). The forces exposed in our paper, where agents coordinate on common pieces of

The mechanism in our paper can also be linked to the vast herding literature that followed Scharfstein and Stein (1990). In their model, career-concerned managers make investment decisions sequentially. They tend to follow the decisions of their predecessors, wishing to convey to the public that their information is correlated with the information of others, and thus is likely to be of high quality. Our mechanism is different since it does not rely on career concerns, and because traders act simultaneously without observing what other traders do. Finally, in a concurrent and independent paper, Angeletos, Lorenzoni, and Pavan (2007) analyze how learning by Wall Street traders from aggregate investments of “Silicon Valley” firms can lead to informational complementarities. Our paper is different from theirs in the context of the study and in the modeling device. Hence, the two papers yield different results and implications.

The remainder of this paper is organized as follows. In Section 2, we present the model setup. Section 3 characterizes the equilibrium of the model. In Section 4, we describe the notion of informational complementarities that emerges in our model and how it leads to currency attacks. Section 5 analyzes the effectiveness of learning by the central bank from market outcomes, and shows how non-fundamental volatility emerges in exchange rates and policy making. In Section 6, we analyze the desirability of two policy tools in our model. Section 7 concludes.

## 2 The Model Setup

### 2.1 Payoffs

The players in our model are a central bank and a continuum of currency speculators. Initially, there is a currency peg in place. The central bank has to make a decision whether to maintain the currency peg or not. The value of maintaining the currency peg is characterized by a random state of the fundamental  $\theta$ . This fundamental may represent the terms of trade, the level of productivity in the economy, or the state of the banking system, as all these variables determine the prospects of the domestic currency and hence the desirability of maintaining the peg at the existing level (see discussion in the introduction).

The regime outcome is given by  $\delta \in \{0, 1\}$  where  $\delta = 1$  indicates that the central bank

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information due to informational complementarities may shed new light on the problems studied in this literature.

defends the status quo and  $\delta = 0$  indicates that the central bank abandons the status quo. The regime outcome  $\delta$  is controlled by the central bank, whose payoff is given by

$$U = \delta\theta. \tag{1}$$

Clearly, if the central bank was perfectly informed about the fundamental, the optimal decision would be to set  $\delta = 1$  if  $\theta > 0$  and set  $\delta = 0$  if  $\theta < 0$ . In reality, the central bank is likely to be imperfectly informed, and our analysis focuses on the central bank's exchange rate policy in this case.

A continuum of speculators of measure one, indexed by  $i$  and uniformly distributed over  $[0, 1]$ , decide whether to short-sell the currency (i.e., attack the regime) or not. We assume that speculators are wealth constrained and can only short-sell up to one unit of the currency. The payoff of a speculator who does not attack is normalized to zero. The payoff from attacking the currency is  $1 - c$  if the status quo is abandoned (i.e., if the central bank sets  $\delta = 0$ ) and  $-c$  otherwise. Here,  $c \in (0, 1)$  is the opportunity cost of attacking.<sup>13</sup>

## 2.2 Timing

The central bank and the speculators play the following game. First, both the central bank and the speculators receive information regarding  $\theta$ . Then, the speculators independently and simultaneously decide whether to attack the currency or not. Finally, after observing the size of the aggregate attack from speculators, the central bank decides whether to maintain the status quo or not. Note that, unlike in the existing literature, the size of the speculative attack does not enter the central bank's payoff function in Equation (1). In our model, the effect of the speculative attack is due to the information revealed by the attack about the realization of the fundamental  $\theta$ .

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<sup>13</sup>To simplify the algebra we follow some of the recent literature and assume that the payoff for speculators in case of devaluation is fixed. Alternatively, we could model the exchange rate determination by allowing the central bank to devalue the currency to the expected value of the fundamental, in which case the speculators' payoff would depend on the fundamental. This alternative setup would not change the basic forces of our model.

## 2.3 Information

We assume that the central bank and the speculators have a common prior about  $\theta$  which is an improper uniform over  $\mathbb{R}$ . The central bank receives a private signal  $s_b = \theta + \sigma_b \epsilon_b$  about the fundamental, where  $\epsilon_b$  is standard normally distributed (i.e., with mean of zero and standard deviation of one). We denote the p.d.f. and the c.d.f. of the standard normal distribution by  $\phi$  and  $\Phi$ , respectively. We denote the precision of the central bank's signal by  $\tau_b = 1/\sigma_b^2$ . The central bank also observes the size of the attack  $A$  from speculators. To reduce notational complexity, we use the variable  $T \equiv \Phi^{-1}(A)$  instead of  $A$  in analyzing the model.

We assume that speculator  $i \in [0, 1]$  receives two signals about  $\theta$  that are both privately observed. One signal,  $s_i$ , is conditionally independent across speculators and the other,  $s_{pi}$ , has a noise component that is commonly shared by all speculators and thus is correlated across speculators conditional on  $\theta$ . More specifically,  $s_i$  is of the form  $s_i = \theta + \sigma_s \epsilon_i$  where  $\epsilon_i$  is normally distributed with mean of zero and standard deviation of one, and the precision of the signal is denoted by  $\tau_s = 1/\sigma_s^2$ . The second signal  $s_{pi}$  is of the form  $s_{pi} = \theta + \sigma_p \epsilon_p + \sigma_h \eta_i$  where  $\epsilon_p$  and  $\eta_i$  are both normally distributed with mean of zero and standard deviation of one. We let  $\tau_p = 1/\sigma_p^2$ , and  $\tau_h = 1/\sigma_h^2$ , so that the precision of  $s_{pi}$  is  $\tau'_p = \tau_h \tau_p / (\tau_h + \tau_p)$ . The signals  $s_{pi}$  share a common noise term  $\epsilon_p$ , and hence are correlated across all speculators conditional on  $\theta$ . This information structure is motivated by the notion that a part of the information generating process may be subject to common random shocks such as market-wide rumors.<sup>14</sup> Notice that we are not assuming that speculators individually are better informed than the central bank. In fact, each speculator's information might be much noisier than the central bank's information. The payoff and information structure are common knowledge. All error terms –  $\epsilon_b$ ,  $\epsilon_i$ ,  $\epsilon_p$ , and  $\eta_i$  – are independent of each other and  $\epsilon_i$  and  $\eta_i$  are independent across investors.

## 3 Equilibrium

We now formally define an equilibrium in this setting. Let  $g(s_i, s_{pi})$  denote the action of a speculator given signals  $s_i$  and  $s_{pi}$ ,  $T(\theta, \epsilon_p)$  the size of the aggregate attack from speculators

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<sup>14</sup>Allowing the central bank to observe a signal like  $s_{pi}$  does not change our results but adds significant complexity to the derivations.

for given fundamental  $\theta$  and the common noise term  $\epsilon_p$ , and  $\delta(T, s_b)$  the action of the central bank as a function of the size of the attack and its signal. Furthermore, let  $\nu(\theta|T, s_b)$  denote the posterior distribution of  $\theta$  conditional on the central bank's information, and let  $\mu(\theta|s_i, s_{pi})$  denote the posterior distribution of  $\theta$  conditional on a speculator's information.

**Definition 1** *An equilibrium consists of a strategy for the central bank,  $\delta(T, s_b)$ , a symmetric strategy for the speculators,  $g(s_i, s_{pi})$ , probability measures,  $\nu(\cdot|T, s_b)$  and  $\mu(\cdot|s_i, s_{pi})$ , such that*

$$\begin{aligned} \delta(T, s_b) &\in \operatorname{argmax}_{\delta \in \{0,1\}} \int_{-\infty}^{\infty} \delta \theta d\nu(\theta|T, s_b), \\ g(s_i, s_{pi}) &\in \operatorname{argmax}_{a \in \{0,1\}} a \cdot \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{1}_{[\delta(T(\theta, \epsilon_p), \theta + \sigma_b \epsilon_b) = 0]} d\mu(\theta|s_i, s_{pi}) d\Phi(\epsilon_b) - c \right], \\ T(\theta, \epsilon_p) &= \Phi^{-1} \left( \int_{-\infty}^{\infty} \left( \int_{-\infty}^{\infty} g(\theta + \sigma_s \epsilon, \theta + \sigma_p \epsilon_p + \sigma_h \eta) \phi(\eta) d\eta \right) \phi(\epsilon) d\epsilon \right), \\ \nu(\theta|T, s_b) &\text{ is obtained using Bayes' rule for any } T \text{ and } s_b, \\ \mu(\theta|s_i, s_{pi}) &\text{ is obtained using Bayes' rule for any } s_i \text{ and } s_{pi}. \end{aligned}$$

Our focus will be on linear threshold equilibria. These are equilibria where speculators attack the currency if and only if the independent signal  $s_i$  is below a threshold  $\hat{s}(s_{pi})$ , which is a linear function of the correlated signal  $s_{pi}$ . In addition, the central bank abandons the regime if and only if the aggregate size of the attack  $T$  is above a threshold  $\hat{T}(s_b)$ , which is also a linear function of its private signal  $s_b$ . The next proposition shows that there is a unique such equilibrium and characterizes it.

**Proposition 1** *There is a unique linear threshold equilibrium where the speculators' threshold strategy is:*

$$g(s_i, s_{pi}) = \begin{cases} 1 & \text{if } s_i \leq \hat{s}(s_{pi}) \\ 0 & \text{if } s_i > \hat{s}(s_{pi}) \end{cases},$$

*and the central bank's strategy is*

$$\delta(T, s_b) = \begin{cases} 1 & \text{if } T \leq \hat{T}(s_b) \\ 0 & \text{if } T > \hat{T}(s_b) \end{cases}.$$

*Here,*

$$\hat{s}(s_{pi}) = \hat{s}(0) - \hat{k}s_{pi}, \tag{2}$$

where  $\hat{k} > 0$  is the unique real root to the cubic equation:

$$\left(-\left(\frac{\tau_s}{\tau_p}\right)(\tau_h + \tau_p)\hat{k}^3 + \tau_h\hat{k}^2\right)\tau_b + \tau_p\left(\hat{k}^2 + 2\hat{k} + 1\right)\left(\tau_h - \tau_s\hat{k}\right) = 0. \quad (3)$$

The threshold  $\hat{s}(0)$  is uniquely determined given the unique  $\hat{k}$ . The threshold value in the central bank's strategy is

$$\hat{T}(s_b) = \frac{1}{\sqrt{\sigma_s^2 + \hat{k}^2\sigma_h^2}} \left[ \hat{s}(0) + \left(1 + \hat{k}\right) \frac{\tau_b}{\tau_T} s_b \right] \quad (4)$$

where

$$\tau_T = \tau_p \left(1 + \frac{1}{\hat{k}}\right)^2 \quad (5)$$

is the precision of the attack as a signal of the fundamental.

#### PROOF OF PROPOSITION 1:

Suppose an agent attacks if and only if  $s_i + \hat{k}s_{pi} \leq \hat{s}(0)$  where  $\hat{k} > 0$ . The size of the attack from speculators given  $\theta$  and  $\epsilon_p$  is  $A(\theta, \epsilon_p) = \Phi\left(\frac{\hat{s}(0) - \hat{k}\sigma_p\epsilon_p - (1 + \hat{k})\theta}{\sqrt{\sigma_s^2 + \hat{k}^2\sigma_h^2}}\right)$ . The central bank observes  $T(\theta, \epsilon_p) = \Phi^{-1}(A)$ , or equivalently, it observes

$$T = \frac{\hat{s}(0) - \hat{k}\sigma_p\epsilon_p - (1 + \hat{k})\theta}{\sqrt{\sigma_s^2 + \hat{k}^2\sigma_h^2}}, \quad (6)$$

which can be rewritten as

$$\frac{\hat{s}(0) - \sqrt{\sigma_s^2 + \hat{k}^2\sigma_h^2}T}{1 + \hat{k}} = \theta + \frac{\hat{k}\sigma_p}{1 + \hat{k}}\epsilon_p.$$

Thus, the precision of the attack as a signal of the fundamental is

$$\tau_T = \tau_p \left(1 + \frac{1}{\hat{k}}\right)^2,$$

and the expected fundamental given the central bank's information is

$$E[\theta|T, s_b] = \frac{\tau_T}{\tau_T + \tau_b} \left( \frac{\hat{s}(0) - \sqrt{\sigma_s^2 + \hat{k}^2\sigma_h^2}T}{1 + \hat{k}} \right) + \frac{\tau_b}{\tau_T + \tau_b} s_b.$$

This implies the status quo is abandoned if and only if

$$T \geq \frac{\hat{s}(0)}{\sqrt{\sigma_s^2 + \hat{k}^2\sigma_h^2}} + \frac{(1 + \hat{k})}{\sqrt{\sigma_s^2 + \hat{k}^2\sigma_h^2}} \frac{\tau_b}{\tau_T} s_b = \hat{T}(s_b),$$

which is Equation (4).

The posterior belief of regime change for a speculator with signals  $s_i$  and  $s_{pi}$  can then be expressed as:

$$\Pr \left( T \geq \frac{\hat{s}(0)}{\sqrt{\sigma_s^2 + \hat{k}^2 \sigma_h^2}} + \frac{(1 + \hat{k})}{\sqrt{\sigma_s^2 + \hat{k}^2 \sigma_h^2}} \frac{\tau_b}{\tau_T} s_b | s_i, s_{pi} \right).$$

Plugging  $T$  from (6), this probability can be rewritten as:

$$\Pr \left( (1 + (1 + \hat{k}) \frac{\tau_b}{\tau_T}) \theta + (1 + \hat{k}) \frac{\tau_b}{\tau_T} \sigma_b \epsilon_b - \hat{k} \sigma_h \eta_i \leq -\hat{k} s_{pi} | s_i, s_{pi} \right).$$

For a speculator,  $\theta$  is distributed with mean  $\frac{\tau_s}{\tau_s + \tau'_p} s_i + \frac{\tau'_p}{\tau_s + \tau'_p} s_{pi}$ , where  $\tau'_p = \tau_h \tau_p / (\tau_h + \tau_p)$ , and  $\sigma_h \eta_i$  is distributed with mean  $\frac{\tau_p \tau_s}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s} (s_{pi} - s_i)$ . Let  $\Omega$  be the standard deviation of  $\left[ (1 + (1 + \hat{k}) \frac{\tau_b}{\tau_T}) \theta + (1 + \hat{k}) \frac{\tau_b}{\tau_T} \sigma_b \epsilon_b - \hat{k} \sigma_h \eta_i \right]$ . We can then express the posterior belief of regime change for a speculator with signals  $s_i$  and  $s_{pi}$  as:

$$\Phi \left( \frac{-\hat{k} s_{pi} - (1 + (1 + \hat{k}) \frac{\tau_b}{\tau_T}) \left( \frac{\tau_s}{\tau_s + \tau'_p} s_i + \frac{\tau'_p}{\tau_s + \tau'_p} s_{pi} \right) + \hat{k} \frac{\tau_p \tau_s}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s} (s_{pi} - s_i)}{\Omega} \right).$$

A speculator would attack only if the cost of attacking  $c$  is smaller than this probability.

After rearranging the above expression, we get the following condition for attack:

$$c \leq \Phi \left( \left( \begin{aligned} & \left( -(1 + (1 + \hat{k}) \frac{\tau_b}{\tau_T}) \frac{\tau_s}{\tau_s + \tau'_p} - \hat{k} \frac{\tau_p \tau_s}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s} \right) s_i \\ & + \left( -\hat{k} - (1 + (1 + \hat{k}) \frac{\tau_b}{\tau_T}) \frac{\tau'_p}{\tau_s + \tau'_p} + \hat{k} \frac{\tau_p \tau_s}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s} \right) s_{pi} \end{aligned} \right) / \Omega \right). \quad (7)$$

Condition (7) can be written as:

$$s_i + B(\hat{k}) s_{pi} \leq - \frac{\Phi^{-1}(c) \Omega}{(1 + (1 + \hat{k}) \frac{\tau_b}{\tau_T}) \frac{\tau_s}{\tau_s + \tau'_p} + \hat{k} \frac{\tau_p \tau_s}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s}}, \quad (8)$$

where

$$\begin{aligned} B(\hat{k}) &\equiv \frac{\hat{k} + (1 + (1 + \hat{k}) \frac{\tau_b}{\tau_T}) \frac{\tau'_p}{\tau_s + \tau'_p} - \hat{k} \frac{\tau_p \tau_s}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s}}{(1 + (1 + \hat{k}) \frac{\tau_b}{\tau_T}) \frac{\tau_s}{\tau_s + \tau'_p} + \hat{k} \frac{\tau_p \tau_s}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s}} \\ &= \frac{\tau_h (1 + \frac{\tau_b \hat{k}^2}{\tau_p (1 + \hat{k})}) \tau_p + \hat{k} (\tau_s + \tau_p)}{\tau_s (1 + \frac{\tau_b \hat{k}^2}{\tau_p (1 + \hat{k})}) (\tau_p + \tau_h) + \hat{k} \tau_p}. \end{aligned} \quad (9)$$

In the proposed linear equilibrium, we should have  $B(\hat{k}) = \hat{k}$ . This implies that  $\hat{k}$  is a root of  $H(k) = 0$ , where:

$$H(k) = \left( - \left( \frac{\tau_s}{\tau_p} \right) (\tau_h + \tau_p) k^3 + \tau_h k^2 \right) \tau_b + \tau_p (k^2 + 2k + 1) (\tau_h - \tau_s k).$$

Next, we show that there is a unique root for  $H(k) = 0$  which is strictly positive. Consider the following equation:

$$H_a(k) = -(\tau_h + \tau_p) \left( \frac{\tau_s \tau_b}{\tau_p} \right) k^3 + (\tau_b + \tau_p) \tau_h k^2 + 2\tau_p \tau_h k + \tau_p \tau_h. \quad (10)$$

Compute the discriminant for  $H_a(k)$ :

$$\begin{aligned} \Delta = & -4(\tau_b + \tau_p)^3 \tau_h^3 \tau_p \tau_h + (\tau_b + \tau_p)^2 \tau_h^2 (2\tau_p \tau_h)^2 + 4(\tau_h + \tau_p) \left( \frac{\tau_s \tau_b}{\tau_p} \right) (2\tau_p \tau_h)^3 \\ & - 18(\tau_h + \tau_p) \left( \frac{\tau_s \tau_b}{\tau_p} \right) (\tau_b + \tau_p) \tau_h (2\tau_p \tau_h) \tau_p \tau_h - 27(\tau_h + \tau_p)^2 \left( \frac{\tau_s \tau_b}{\tau_p} \right)^2 (\tau_p \tau_h)^2. \end{aligned}$$

It is straightforward to verify that  $\Delta < 0$ , so  $H_a(k) = 0$  must have a unique real root.  $H_a(k)$  goes to  $\infty$  as  $k$  goes to  $-\infty$ , goes to  $-\infty$  as  $k$  goes to  $\infty$ , and  $H_a(0) > 0$ . These facts and the fact that the equation has a single real root implies that it must cross zero at a unique  $k > 0$ .

Note that

$$H(k) = H_a(k) - \tau_s \tau_p k(k+1)^2. \quad (11)$$

For  $k < 0$ ,  $H(k) > H_a(k)$ , thus  $H(k) = 0$  cannot have a negative root. Also  $H(0) = H_a(0) > 0$ , and for  $k > 0$ ,  $H(k) < H_a(k)$ . Moreover  $H(k)$  decreases faster than  $H_a(k)$  at all  $k > 0$ . Since  $H_a(k)$  crosses zero only once,  $H(k)$  must also cross zero only once, for some  $k > 0$ . Letting  $\hat{k}$  be the unique root of  $H(k) = 0$  completes the proof. ■

To summarize, in equilibrium, the optimal strategy for a speculator who receives a signal  $s_i$  is to attack if and only if  $s_i$  falls below a threshold value,  $\hat{s}(s_{pi})$ , which is decreasing in the correlated signal  $s_{pi}$ . That is, when the correlated signal indicates a sound fundamental, speculators attack only if their independent private signals are very pessimistic. The weight  $\hat{k}$  that the speculator puts on the correlated signal is derived endogenously. For the central bank, the attack provides an additional signal about the fundamental. This signal has precision  $\tau_T$  which is decreasing in the weight  $\hat{k}$  that speculators put on the correlated signal. The optimal strategy for the central bank is to abandon the exchange rate regime if and only if the observed signal of aggregate attack,  $T$ , is greater than or equal to the threshold,  $\hat{T}(s_b)$ , which is increasing in the central bank's private signal  $s_b$ .

## 4 Informational Complementarities and Currency Attacks

An important element of our equilibrium is  $\hat{k}$  – the weight that speculators put on the correlated signal  $s_{pi}$  in their decision whether to participate in a speculative attack. At a basic level, speculators put a positive weight on  $s_{pi}$  because it provides additional information about the realization of  $\theta$ , and thus on the probability that the central bank will abandon the regime. Because speculators know that the central bank is going to use the information conveyed by the size of the attack in its policy decision, however, they end up putting too much weight on  $s_{pi}$ .

To see this, let us compare the weight  $\hat{k}$  that speculators put on the correlated signal  $s_{pi}$  (implicitly defined in Equation (3)) with a benchmark level  $k_{BM}$  that would be obtained if the central bank did not use the size of the attack to infer information about the fundamental  $\theta$ . The next proposition shows that  $\hat{k}$  – defined in Equation (3) – is indeed greater than  $k_{BM}$ .

**Proposition 2** *The weight  $\hat{k}$  put by speculators on  $s_{pi}$  in the unique linear threshold equilibrium characterized by Proposition 1 is greater than the weight  $k_{BM}$  that would be put on  $s_{pi}$  in a game where the central bank does not attempt to get information about  $\theta$  from the size of the attack.*

### PROOF OF PROPOSITION 2:

In a linear threshold equilibrium, speculators attack if and only if the independent signal  $s_i$  is below the threshold  $\hat{s}_{BM}(s_{pi}) = \hat{s}_{BM}(0) - k_{BM}s_{pi}$ . We now compute the weight  $k_{BM}$  that speculators put on the correlated signal. Since the central bank does not update its belief about  $\theta$  based on the size of the attack, it will abandon the regime if and only if its private signal  $s_b = \theta + \sigma_b\epsilon_b$  is negative. Then, speculators will attack the currency if and only if  $\Pr(\theta + \sigma_b\epsilon_b < 0 | s_i, s_{pi}) \geq c$ . This implies that a speculator observing  $s_i$  and  $s_{pi}$  will attack when:

$$\Phi\left(\frac{-\frac{\tau_s}{\tau_s+\tau'_p}s_i - \frac{\tau'_p}{\tau_s+\tau'_p}s_{pi}}{\sqrt{\frac{1}{\tau_s+\tau'_p} + \sigma_b^2}}\right) \geq c.$$

This can be rewritten as:

$$s_i + \frac{\tau'_p}{\tau_s}s_{pi} \leq -\Phi^{-1}(c) \sqrt{\frac{1}{\tau_s + \tau'_p} + \sigma_b^2}. \quad (12)$$

It follows that:

$$k_{BM} = \frac{\tau'_p}{\tau_s}.$$

Now, we show that  $\hat{k}$  (defined in Equation (3)) is greater than  $k_{BM}$ . Recall that  $\hat{k}$  is determined such that  $B(\hat{k}) = \hat{k}$ . Inspecting equation (9), we can see that  $B(k) > \frac{\tau'_p}{\tau_s} = \frac{\tau_h \tau_p}{(\tau_h + \tau_p) \tau_s}$  for any  $k > 0$ . Since  $\hat{k} > 0$ , it follows that  $\hat{k} > k_{BM}$ . ■

Intuitively, when the central bank does not learn from the size of the attack, there is no strategic interaction among the speculators. As a result, the weight that they put on the correlated signal  $s_{pi}$  depends only on the informativeness of this signal, relative to the informativeness of the independent signal  $s_i$ , about the fundamental  $\theta$  (which is correlated with the central bank's private signal and its policy decision). The result is a benchmark weight of  $k_{BM} = \tau'_p / \tau_s$ . When the central bank learns from the size of the attack, strategic interactions arise and lead to a higher weight  $\hat{k}$ .

For illustration, consider equations (8) and (9) in the proof of Proposition 1. Equation (8) is the derived decision rule of a speculator on when to attack the regime in the full model (where the central bank learns from the size of the attack), given that other speculators play the proposed equilibrium strategy. Function  $B(k)$  can then be thought of as the best response of a speculator to other speculators' weight on the correlated signal. That is, if all speculators in the economy put a relative weight  $k$  on the correlated signal when deciding whether to attack or not, the best response for a speculator is to put the weight  $B(k)$  on his correlated signal. The symmetric equilibrium is solved when the best response crosses the 45-degree line; i.e., when  $\hat{k} = B(\hat{k})$ .<sup>15</sup> The response function in the benchmark model (where the central bank does not learn from the size of the attack) can be denoted as  $B_{BM}(k) \equiv \tau'_p / \tau_s$ . In this case, the weight that a speculator puts on the correlated signal is independent of the weight chosen by others and is equal to the informativeness ratio  $\tau'_p / \tau_s$ . The two functions and the corresponding equilibrium weights are plotted in the following figure.

As we can see in the figure,  $B(0) = B_{BM}(0) = \tau'_p / \tau_s$ . That is, when other speculators put no weight on the correlated signal, each speculator finds it optimal to put the benchmark weight  $\tau'_p / \tau_s$  on this signal. In this case, because other speculators do not use the correlated signals, an individual speculator knows that both  $s_i$  and  $s_{pi}$  do not provide any information

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<sup>15</sup>This is the logic used in the proof of Proposition 1. We thank one of the referees for pointing out this exposition.

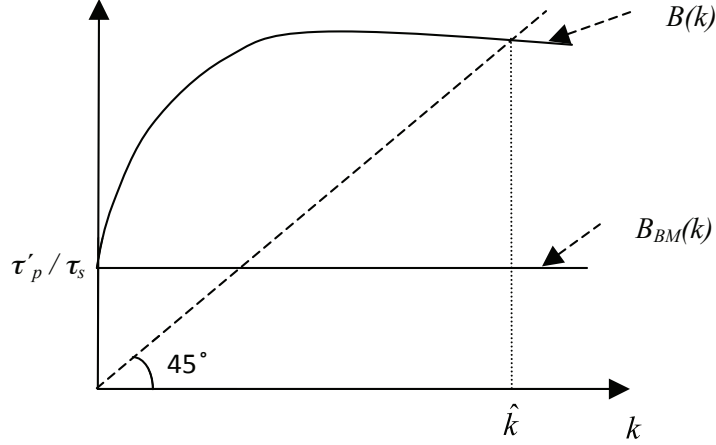


Figure 1: Best Response:  $B(k)$  and  $B_{BM}(k)$

about the attack and the resulting central bank's action beyond the information they provide about  $\theta$ . Hence, the speculator puts weights on these signals based on their relative informativeness.

Once  $k$  increases above 0, strategic complementarities emerge in the full model, and  $B(k)$  starts increasing above  $\tau'_p / \tau_s$ . As other speculators put more weight on the correlated signal, an individual speculator knows that his correlated signal provides additional information about the size of the attack beyond the information about  $\theta$ . Then, since the central bank is more likely to believe that  $\theta$  is low and abandon the regime when it sees a large attack, the speculator wishes to act like other speculators and puts more weight on  $s_{pi}$ . Note that unlike in Morris and Shin (1998, 2002), strategic complementarities here emerge endogenously as a result of learning, and are not directly imposed on the payoff functions. We thus distinguish the *informational complementarities* in our setting from the *direct complementarities* in the previous literature.

As the figure shows, for a large  $k$ ,  $B(k)$  may eventually start decreasing. A formal analysis of  $B(k)$  (the expression for which is given in equation (9)) shows that this only happens when  $\tau_p < \tau_b$  and  $k > \frac{1}{\tau_b - \tau_p} (\tau_p + \sqrt{\tau_b \tau_p})$ . Intuitively, as  $k$  gets very large, the central bank puts little weight on the attack in its policy decision, in which case the incentive to coordinate decreases, leading speculators to revert the weight they put on the correlated

signal towards the benchmark weight (but not all the way). Note that this happens only when the central bank's information is more precise than the common component in the speculators' correlated signals. Otherwise, the weight put by the central bank on the attack when  $k$  is large is still sufficiently high, and so  $B(k)$  continues to increase (indeed, when  $\tau_p \geq \tau_b$ ,  $B(k)$  is monotonically increasing in  $k$ ).

Overall, as shown in the proof of Proposition 2 and demonstrated in the figure,  $\hat{k} > k_{BM}$ . Hence, informational complementarities are the dominant force generated by the fact that the central bank learns from the attack. Informational complementarities create coordination motives that lead speculators to put more weight on their correlated signals. As a result, our model generates large currency attacks that are not justified by fundamentals. Essentially, when  $\epsilon_p$  is low, a large currency attack will be formed despite the fundamentals being reasonably high. Since the central bank does not observe the common noise component  $\epsilon_p$  that drives the correlation (recall that  $\epsilon_p$  is not observed by anyone in the economy), it does not know if a large attack is due to *coordination* or *information*, and so it can be “fooled” to take the wrong action. This justifies the equilibrium strategy of speculators to put excess weight on the correlated signal.

More generally, our results require the speculators to be able to coordinate their actions through some commonality of their information, which is relevant for the central bank's decision, but the central bank cannot fully tease out. We develop a variant of our model with an alternative information structure in Appendix B where we show that a very similar result holds even when the speculators receive a signal which is not fundamental related. Specifically, we assume that the central bank observes the size of the attack with noise and the speculators are able to observe the noise component of the size of the attack. This assumption captures the idea that speculators may share some common information regarding random shocks to the institutional environment or to the workings of the currency market, which are not known to the central bank (for example, traders may know other traders personally, and thus expect changes in their appetite for risk).

In the next sections, we develop the main implications of these complementarities in our framework, concerning the effectiveness of learning from the market, non-fundamental volatility, time-consistency, and transparency in exchange-rate policy. Before turning to the next section, we now derive comparative statics results on the determinants of the equilibrium weight,  $\hat{k}$ .

**Proposition 3** *The equilibrium weight on the correlated signal,  $\hat{k}$ , decreases in  $\tau_b$  and  $\tau_s$ , and increases in  $\tau_h$  and  $\tau_p$ .*

PROOF OF PROPOSITION 3: See Appendix A.

In other words, if the central bank's signal or the speculators' independent signals are less precise, or their correlated signals are more precise (due to higher  $\tau_h$  and/or  $\tau_p$ ), speculators coordinate better in equilibrium so that the weight they put on the correlated signal increases. Intuitively, if the central bank holds a precise signal, it relies less on the information revealed in the aggregate actions of the speculators. In equilibrium, this forces the speculators to reduce the weight they put on the correlated signal and to reveal more information to the central bank. If each speculator holds a very sharp independent signal about the fundamental, each bases the decision to attack mostly on this signal rather than the noisy correlated signal, and hence there is less incentive to coordinate. Finally, the incentive to coordinate is largest when the correlated signal is very precise. In this case, the speculators put a larger weight on the correlated signal and the central bank cannot ignore the information revealed in the aggregate attack.

## 5 The Effectiveness of Learning from the Market

We now analyze how informational complementarities impact the effectiveness of learning from the market. We do this by looking at the probability of policy mistakes. The central bank makes a policy mistake in our model when it abandons (maintains) the status quo given that  $\theta > 0$  ( $\theta < 0$ ). The following proposition characterizes the probability of making a policy mistake and studies its properties.

**Proposition 4** *The ex ante probability of abandoning the status quo for a given  $\theta$  is*

$$\Phi(-\sqrt{\tau_b + \tau_T}\theta). \quad (13)$$

*Hence, when  $\theta > 0$ , the probability of making a policy mistake is  $\Phi(-\sqrt{\tau_b + \tau_T}\theta)$ , while when  $\theta < 0$ , it is  $1 - \Phi(-\sqrt{\tau_b + \tau_T}\theta)$ . In both cases, the probability of making a policy mistake decreases in  $\tau_T$  – the precision of the signal provided by the attack to the central bank.*

#### PROOF OF PROPOSITION 4:

The *ex ante* probability of abandoning the status quo given  $\theta$  is:

$$\Pr \left( T \geq \frac{\hat{s}(0)}{\sqrt{\sigma_s^2 + \hat{k}^2 \sigma_h^2}} + \frac{(1 + \hat{k})}{\sqrt{\sigma_s^2 + \hat{k}^2 \sigma_h^2}} \frac{\tau_b}{\tau_T} s_b | \theta \right) = \Pr \left( \epsilon_b + \frac{\sqrt{\tau_T}}{\sqrt{\tau_b}} \epsilon_p \leq -\frac{1}{\sigma_b} \left( 1 + \frac{\tau_T}{\tau_b} \right) \theta | \theta \right)$$

where the equality follows by plugging in for  $T$  (see (6)),  $\tau_T$  (see (5)), and  $s_b$  and rearranging. The term  $\epsilon_b + \frac{\sqrt{\tau_T}}{\sqrt{\tau_b}} \epsilon_p$  is the weighted sum of two independent normal random variables, so this term itself is normal with mean 0 and variance  $1 + \frac{\tau_T}{\tau_b}$ . Then, it is easy to show that the ex-ante probability of abandoning the status quo for a given  $\theta$  is:

$$\Pr \left( \epsilon_b + \frac{\sqrt{\tau_T}}{\sqrt{\tau_b}} \epsilon_p \leq -\frac{1}{\sigma_b} \left( 1 + \frac{\tau_T}{\tau_b} \right) \theta | \theta \right) = \Phi \left( -\sqrt{\tau_b + \tau_T} \theta \right).$$

The rest of the proposition follows directly. ■

Intuitively, the probability of making a policy mistake decreases in the precision of the two pieces of information that the central bank has: the precision of the information conveyed by the size of the attack  $\tau_T$  and the precision of the central bank's private information  $\tau_b$ . Since  $\tau_T$  is decreasing in the weight  $\hat{k}$  that speculators put on the correlated signal (see (5)), this weight has a positive effect on the probability of a policy mistake. Hence, informational complementarities that lead speculators to coordinate their actions and put a higher weight on their correlated signals, increase the probability that the central bank will make a policy mistake.

The interesting implication of this analysis is that learning from the market has a self-defeating aspect. In Proposition 2, we showed that by learning from the market, the central bank gives rise to informational complementarities that increase the weight  $\hat{k}$  that speculators put on the correlated signal. Now we see that  $\hat{k}$  increases the probability of policy mistakes. Hence, when using the information from the market, the central bank reduces the quality of this information. It should be noted that informational complementarities are not the only source of policy mistakes in the model. As long as the precision of the correlated signal is non-zero, speculators will put a positive weight on this signal (see the definition of  $k_{BM}$  above), and so the common noise in the correlated signal will be reflected in the size of the attack and generate policy mistakes. Yet, the presence of complementarities – generated by the central bank's learning – amplifies the weight speculators put on the correlated signal, and increases the tendency for policy mistakes.

Overall, we see that the presence of correlated signals reduces the effectiveness of learning from the market. Consider what would happen in a model where the correlated signal played no role. To analyze this formally, we let  $\tau_p$  approach zero. As the next proposition shows, in this case, the speculators can no longer coordinate on the correlated signal, i.e.,  $\hat{k} \rightarrow 0$ . Then, the attack becomes fully revealing of the fundamental, as the noise terms of the idiosyncratic signals cancel out with each other, and the central bank does not make policy mistakes.<sup>16</sup>

**Proposition 5** *In the limit as  $\tau_p$  approaches 0,  $\hat{s}(s_{pi}) = \hat{s}(0) = \sigma_s \Phi^{-1}(1 - c)$ , and the attack becomes fully revealing of  $\theta$ .*

PROOF OF PROPOSITION 5: See Appendix A.

Taken together, the two propositions above generate an intriguing result. Even though speculators are better informed about the fundamental when they observe a correlated signal in addition to their private signals, the central bank, paradoxically, becomes less informed when this signal is introduced. This suggests that adding a correlated source of information reduces the effectiveness of the learning process for the central bank. We now explore this aspect of the model more fully by analyzing the effect that the precision of the various signals has on the effectiveness of learning from the market. The following proposition provides the main result.

**Proposition 6** *Conditional on  $\theta$ , the ex-ante probability of making a policy mistake decreases in  $\tau_b$  and  $\tau_s$ , increases in  $\tau_p$  if  $0 < \tau_p < \bar{\tau}_p$  and decreases in  $\tau_p$  if  $\tau_p > \bar{\tau}_p$  where  $\bar{\tau}_p = \frac{1}{8} \frac{\tau_h^2 \tau_b}{(\tau_h + \tau_s)^2} \left( \sqrt{1 + \frac{16\tau_s}{\tau_b} + \frac{16\tau_s^2}{\tau_h \tau_b}} - 1 \right)$ .*

PROOF OF PROPOSITION 6: See Appendix A.

The results regarding  $\tau_s$  and  $\tau_b$  are straightforward. Improving the precision of the speculators' independent signals or the central bank's signal generates a decrease in the probability that the central bank will make a policy mistake. But the effect of increasing the precision of the correlated signal by increasing  $\tau_p$  can go in both directions. On the one hand, increasing the precision of the correlated signal implies that speculators have access to

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<sup>16</sup>In Appendix C, we study the other limit where the speculators commonly know the fundamental. We show that in this limit the linear equilibrium disappears, and there are multiple equilibria. In these equilibria, speculators coordinate their actions perfectly by either all of them or none of them attacking for a given value of the fundamental.

more precise information, which can be revealed to the central bank via the trading process. On the other hand, increasing the precision of the correlated signal implies that speculators will rely more on the correlated signal. That is, the ability of speculators to coordinate on the correlated signal and convey a misleading message to the central bank improves. Our result shows that the first effect dominates when  $\tau_p$  is above a certain cutoff and the second effect dominates when  $\tau_p$  is below the cutoff. Hence, the probability of a policy mistake is maximized at an intermediate level of  $\tau_p$ .

Another way to view our results is that informational complementarities lead to non-fundamental volatility in the exchange rate regime. Due to informational complementarities, the common noise in the correlated signal gets to have a significant impact on speculators' tradings and leads to many instances where the exchange rate regime is abandoned without the fundamentals justifying it. In that, our model rationalizes the empirical results documented by Eichengreen, Rose, and Wyplosz (1995), showing that there are excessive transitions between exchange rate regimes and that these transitions cannot be explained by fundamentals. As implied by our analysis, non-fundamental volatility is expected even without informational complementarities, just because speculators put the benchmark weight  $k_{BM}$  on the correlated signals. The role of informational complementarities is to increase this weight above the benchmark level and hence amplify the non-fundamental volatility. Thus, the empirical advantage of a model with complementarities is that it generates bigger effects.

Policy mistakes may also lead to a rebound in the exchange rate after a speculative attack leads to devaluation. Such rebounds are commonly observed, and led to a large theoretical literature attributing them to multiple equilibria (see Flood and Garber (1984), Obstfeld (1996), and others). Our paper suggests a different mechanism behind such frequent rebounds which is linked to the structure of information in the foreign exchange market. For example, devaluations that follow speculative attacks are more likely to be reversed in an environment where the independent source of information available to speculators is imprecise and the correlated information is moderately precise.

We conclude this section by pointing to another interesting property of speculative attacks in our model, which stands in stark difference to existing models of currency attacks. This is summarized in the next corollary.

**Corollary 1** *The level of the opportunity cost,  $c$  (as well as the wealth level of speculators) affects the size of the attack but does not affect the information content of the attack and,*

hence, does not affect the probability of devaluation occurring for a given  $\theta$ .

The reason for this result is that the central bank does not care about the size of the attack per se when making a policy decision. A large attack will induce the central bank to abandon the exchange rate regime only if it provides information that the fundamentals are low. Hence, while a decrease in the cost of attacking the regime will increase the tendency of speculators to attack, this will be filtered out by the central bank, and will not change the overall tendency to abandon the regime. The implication is that when we compare across markets, some speculative attacks will be defended, while others that are weaker (but provide more information) will not.

## 6 Policy Implications

We now analyze the effect of two policy tools on the effectiveness of learning from the market.

### 6.1 Commitment

The first policy tool that we explore is commitment by the central bank to reduce the weight it puts on the information in the size of the attack below what is ex-post optimal. The next proposition analyzes the overall desirability of such a commitment policy.

**Proposition 7** *Suppose that speculators follow linear strategies, then the central bank can always decrease the probability of making a policy mistake by committing to put a slightly lower (higher) weight on the information in the attack (on its private signal) than is ex-post optimal.*

PROOF OF PROPOSITION 7: See Appendix A.

The proposition says that it is always optimal for the central bank to commit to a slight deviation from the ex-post optimal weights, and in particular to increase the weight given to its private signal and decrease the weight given to the information in the size of the attack. The reason follows from the envelope theorem, as we know that the cost of a small deviation from the ex-post optimal weight approaches 0, but the benefit from the increased informativeness of the attack is always strictly positive. As a result, slight deviations from ex-post optimal weights are always desirable. Of course, large deviations are not desirable. For example, it is never optimal for the central bank to completely ignore the attack as a

signal for the fundamentals. This is because the only advantage in reducing the weight on the attack in the policy decision is the improved information coming from the attack, but this is of no use if the attack is completely ignored.

Proposition 7 exposes a problem of time inconsistency in the central bank's policy. While the central bank wants to commit *ex ante* to put a lower weight on the information coming from speculative attacks, once the attack is realized *ex post*, the central bank would always be tempted to pay more attention to it. In practice, such commitment can be achieved by having an overconfident policymaker, who thinks that his information is more precise than it really is.<sup>17</sup> Another possibility is to design an incentive contract that incentivizes the policymaker to put different weights on the pieces of information than is ex-post optimal.

## 6.2 Transparency

Next, we discuss the issue of central bank transparency, in other words, whether and how clearly the central bank should communicate its information to the public. The issue of central bank transparency is receiving a lot of attention in research and policy circles. The positive aspects of transparency are often emphasized. In a recent paper, Morris and Shin (2005) demonstrate a cost associated with transparency. Building on the insight in Morris and Shin (2002), they show that in the presence of direct complementarities among market participants, transparency can be bad because it provides a public signal, and thus reduces the extent to which speculators use their private information. This, in turn, reduces the ability of the central bank to infer new information from the market.

We analyze the effect of transparency in our framework, where there are no direct complementarities among speculators, but rather speculators care about each other's strategies because they know that their collective action reveals information that affects the central bank's policy decision. We discover a new negative effect of transparency. When the central bank becomes more transparent, it reveals information about the course of action that it is likely to take in the future. Knowing this, the speculators can better coordinate on conveying a misleading signal. To see this, consider the extreme case where the central bank perfectly reveals its information, and suppose that speculators know that solely on the basis

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<sup>17</sup>This result is similar in intuition to that obtained in Bolton, Brunnermeier and Veldkamp (2007). They show that over-confidence is a valuable leadership attribute since it helps leaders to stick to their prior belief when constantly learning about the optimal action in a changing environment.

of this information, the central bank would devalue. In this case, it is an equilibrium for all the speculators to attack. Since the attack reveals no information, the central bank will indeed devalue, and completely miss the opportunity to learn from the market to shape its policy decision. The next proposition analyzes the more interesting case where the central bank releases its information with some noise. We focus on threshold equilibria in which a speculator attacks if and only if his independent signal falls below a threshold that is linear in both the correlated signal and the signal released by the central bank. The proposition shows that as long as the central bank releases its information publicly with enough noise, there is a unique threshold equilibrium. Moreover, in this equilibrium the speculators coordinate more on the correlated information and the central bank is more likely to make a policy mistake.

**Proposition 8** *Suppose that the central bank releases a public signal  $s_a = s_b + \sigma_a \epsilon_a$  where  $\epsilon_a \sim N(0, 1)$ . There is a unique linear threshold equilibrium if  $\sigma_a$  is not too small. In this unique equilibrium, the central bank is more likely to make a policy mistake relative to the equilibrium characterized in Proposition 1 where the central bank releases no information. Moreover, policy mistakes become more likely as the central bank becomes more transparent, i.e., as  $\sigma_a$  becomes smaller.*

PROOF OF PROPOSITION 8: See Appendix A.

The proposition indicates that the central bank may inadvertently strengthen the coordination incentive by releasing more information that becomes common to the speculators. In fact, the more precise such information is, the stronger the coordination among speculators. As discussed above, this result is due to the fact that speculators can coordinate better when they have common information about the central bank's signal, since this information is very revealing about the central bank's action.

Finally, we would like to stress that the negative effect of transparency on policy is due to the fact that the central bank releases information which becomes common knowledge to all speculators. One could imagine a different form of transparency, by which the central bank releases information that is interpreted differently by different speculators. This can be achieved by delivering ambiguous statements that leave room for different interpretations. In such a case, the information conveyed by the central bank provides another source of difference across speculators' information sets, and leads them to coordinate less. Then, the above conclusion is overturned and transparency enables the central bank to learn more effectively

from the attack. Of course, it is more difficult to translate this notion of transparency to practice, since it is not immediately clear how the central bank may send a signal that is interpreted differently by different speculators. The following proposition summarizes the formal result.

**Proposition 9** *Suppose that the central bank releases a public signal, which is interpreted differently by different agents. Specifically, each agent observes  $s_{ai} = s_b + \sigma_a \epsilon_{ai}$  where  $\epsilon_{ai} \sim N(0, 1)$  and drawn independently across speculators. The weight that the speculators put on  $s_{pi}$  in the unique linear threshold equilibrium is smaller than the one characterized in Proposition 1.*

#### PROOF OF PROPOSITION 9:

The central bank observes  $s_b$ , and speculators observe  $s_{ai}$ ,  $s_{pi}$  and  $s_i$ . The precision of  $s_{ai}$  is  $\tau_{ah} = \tau_a \tau_b / (\tau_a + \tau_b)$ . The variance of the speculator's private information about  $\theta$  (that is, two combined private signals) is  $1/(\tau_s + \tau_{ah})$ , smaller than  $1/\tau_s$ . Following steps that are similar to those in the proof of Proposition 1 we find that the equilibrium weight  $k'_{tr}$  put by speculators on  $s_{pi}$  in the unique linear threshold equilibrium satisfies

$$\left( - \left( \frac{\tau_s + \tau_{ah}}{\tau_p} \right) (\tau_h + \tau_p) (k'_{tr})^3 + \tau_h (k'_{tr})^2 \right) \tau_b + \tau_p \left( (k'_{tr})^2 + 2k'_{tr} + 1 \right) (\tau_h - (\tau_s + \tau_{ah}) k'_{tr}) = 0.$$

(There are two differences between the derivations of the equilibrium condition in Proposition 1 and the one above. First,  $\tau_s$  is replaced with  $\tau_s + \tau_{ah}$ . Second, for the speculators  $\theta$  and  $\epsilon_b$  are no longer independently distributed. However, this second difference does not impact the derivation in a substantial way.) The result then follows from Proposition 3. ■

## 7 Conclusion

We analyze a model where the information revealed in the course of a speculative attack is used by the central bank in its policy decisions. On the one hand, this information enhances the effectiveness of the central bank's policy decisions. On the other hand, the fact that the central bank uses the information gives rise to endogenous strategic complementarities – which we call informational complementarities – due to which speculators wish to coordinate on similar trading actions even if they conflict with their private information. These coordination motives reduce the informational content of the speculators' collective action and the effectiveness of the central bank policy decisions. We analyze the tradeoff between

information and coordination in speculative attacks, and derive comparative statics regarding the behavior of speculators and the effectiveness of the central bank's policy decisions. We also analyze the effect of different policy measures that the central bank may adopt to improve the effectiveness of its decisions.

Overall, the contribution of our paper is twofold. First, we introduce an important channel to the literature on currency attacks – namely, the learning by the central bank from market activities. We show the positive and negative aspects of such learning and study the nature of speculative attacks that it generates. Second, we provide a new angle to the literature on the feedback effect from financial markets to the real economy. We show that the fact that a decision maker learns from the trading process and takes an action that affects the value of traded securities gives rise to coordination problems among market participants, which result in destabilizing trading and reduced policy effectiveness. Thus, the analysis here can be used to study other settings where decision makers learn from the aggregate trade in financial markets. Examples include learning by firm managers and providers of capital when deciding whether to go ahead with an investment project.

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## Appendix A

### Proofs

#### PROOF OF PROPOSITION 3

We use the following Lemma in the proof.

**Lemma 1** *The equilibrium weight,  $\hat{k}$ , on the correlated signal  $s_{pi}$  is strictly less than  $\tau_h/\tau_s$ .*

**Proof of Lemma 1** The result follows from the facts that  $H(\tau_h/\tau_s) < 0$  and  $H(k)$  crosses zero from above uniquely at  $\hat{k}$ . ■

To prove the proposition let us first write  $H(k)$  explicitly as a function of  $\tau_h$ ,  $\tau_b$ ,  $\tau_s$  and  $\tau_p$ :

$$H(k; \tau_h, \tau_b, \tau_s, \tau_p) = \left( - \left( \frac{\tau_s}{\tau_p} \right) (\tau_h + \tau_p) k^3 + \tau_h k^2 \right) \tau_b + \tau_p (k^2 + 2k + 1) (\tau_h - \tau_s k).$$

Suppose  $\tau_b > \tau'_b$ . Let  $\hat{k}_b$  be the unique solution to  $H(k; \tau_h, \tau_b, \tau_s, \tau_p) = 0$  and  $\hat{k}'_b$  be the unique solution to  $H(k; \tau_h, \tau'_b, \tau_s, \tau_p) = 0$ . Note that

$$H(k; \tau_h, \tau_b, \tau_s, \tau_p) - H(k; \tau_h, \tau'_b, \tau_s, \tau_p) = (\tau_b - \tau'_b) \left( \left( - \left( \frac{\tau_s}{\tau_p} \right) (\tau_h + \tau_p) k^3 + \tau_h k^2 \right) \right).$$

Since  $H(\hat{k}_b; \tau_h, \tau_b, \tau_s, \tau_p) = 0$ , we have

$$\left( - \left( \frac{\tau_s}{\tau_p} \right) (\tau_h + \tau_p) \hat{k}_b^3 + \tau_h \hat{k}_b^2 \right) \tau_b = -\tau_p (\hat{k}_b^2 + 2\hat{k}_b + 1) (\tau_h - \tau_s \hat{k}_b).$$

Since (by Lemma 1)  $\hat{k}_b < \tau_h/\tau_s$ , we have  $H(\hat{k}_b; \tau_h, \tau_b, \tau_s, \tau_p) < H(\hat{k}_b; \tau_h, \tau'_b, \tau_s, \tau_p)$ . Since both  $H(k; \tau_h, \tau_b, \tau_s, \tau_p)$  and  $H(k; \tau_h, \tau'_b, \tau_s, \tau_p)$  cross zero uniquely and from above the above inequality implies that  $\hat{k}_b < \hat{k}'_b$ . Thus the equilibrium weight on the correlated signal decreases in  $\tau_b$ .

Suppose  $\tau_s > \tau'_s$ . Let  $\hat{k}_s$  be the unique solution to  $H(k; \tau_h, \tau_b, \tau_s, \tau_p) = 0$  and  $\hat{k}'_s$  be the unique solution to  $H(k; \tau_h, \tau_b, \tau'_s, \tau_p) = 0$ . Note that

$$H(k; \tau_h, \tau_b, \tau_s, \tau_p) - H(k; \tau_h, \tau_b, \tau'_s, \tau_p) = (\tau_s - \tau'_s) \left( -\frac{\tau_b}{\tau_p} (\tau_h + \tau_p) k^3 - \tau_p (k^2 + 2k + 1) k \right) < 0.$$

Since  $H(k; \tau_h, \tau_b, \tau_s, \tau_p) < H(k; \tau_h, \tau_b, \tau'_s, \tau_p)$  and both  $H(k; \tau_h, \tau_b, \tau_s, \tau_p)$  and  $H(k; \tau_h, \tau_b, \tau'_s, \tau_p)$  cross zero uniquely and from above the above  $\hat{k}_s < \hat{k}'_s$ . Thus the equilibrium weight on the correlated signal decreases in  $\tau_s$ .

Suppose  $\tau_h > \tau'_h$ . Let  $\hat{k}_h$  be the unique solution to  $H(k; \tau_h, \tau_b, \tau_s, \tau_p) = 0$  and  $\hat{k}'_h$  be the

unique solution to  $H(k; \tau'_h, \tau_b, \tau_s, \tau_p) = 0$ . Note that

$$H(k; \tau_h, \tau_b, \tau_s, \tau_p) - H(k; \tau'_h, \tau_b, \tau_s, \tau_p) = (\tau_h - \tau'_h) \left( \left( -\left( \frac{\tau_s}{\tau_p} \right) k^3 + k^2 \right) \tau_b + \tau_p (k^2 + 2k + 1) \right).$$

Since  $H(\hat{k}_h; \tau_h, \tau_b, \tau_s, \tau_p) = 0$ , we have

$$\left( -\left( \frac{\tau_s}{\tau_p} \right) \hat{k}_h^3 + \hat{k}_h^2 \right) \tau_b \tau_h + \tau_p \tau_h (\hat{k}_h^2 + 2\hat{k}_h + 1) = \left( \frac{\tau_s}{\tau_p} \right) \tau_b \tau_p \hat{k}_h^3 + \tau_p \tau_s (\hat{k}_h^2 + 2\hat{k}_h + 1) \hat{k}_h > 0.$$

Thus,  $H(\hat{k}_h; \tau_h, \tau_b, \tau_s, \tau_p) > H(\hat{k}_h; \tau'_h, \tau_b, \tau_s, \tau_p)$ . Since both  $H(k; \tau_h, \tau_b, \tau_s, \tau_p)$  and  $H(k; \tau'_h, \tau_b, \tau_s, \tau_p)$  cross zero uniquely and from above the above inequality implies that  $\hat{k}_h > \hat{k}'_h$ . Thus the equilibrium weight on the correlated signal increases in  $\tau_h$ .

Finally suppose  $\tau_p > \tau'_p$ . Let  $\hat{k}_p$  be the unique solution to  $H(k; \tau_h, \tau_b, \tau_s, \tau_p) = 0$  and  $\hat{k}'_p$  be the unique solution to  $H(k; \tau_h, \tau_b, \tau_s, \tau'_p) = 0$ . Note that,

$$\begin{aligned} H(k; \tau_h, \tau_b, \tau_s, \tau_p) - H(k; \tau_h, \tau_b, \tau_s, \tau'_p) = \\ \left( \frac{1}{\tau_p} - \frac{1}{\tau'_p} \right) (-\tau_s \tau_h k^3) \tau_b + (\tau_p - \tau'_p) (k^2 + 2k + 1) (\tau_h - \tau_s k). \end{aligned}$$

Since (by Lemma 1)  $\hat{k}_p < \tau_h / \tau_s$ , we have  $H(\hat{k}_p; \tau_h, \tau_b, \tau_s, \tau_p) > H(\hat{k}_p; \tau_h, \tau_b, \tau_s, \tau'_p)$ . Since both  $H(k; \tau_h, \tau_b, \tau_s, \tau_p)$  and  $H(k; \tau_h, \tau_b, \tau_s, \tau'_p)$  cross zero uniquely and from above the above inequality implies that  $\hat{k}_p > \hat{k}'_p$ . Thus the equilibrium weight on the correlated signal increases in  $\tau_p$ . ■

## PROOF OF PROPOSITION 5:

We start with the following lemma:

**Lemma 2** (i)  $\lim_{\tau_p \rightarrow 0} \hat{k}(\tau_p) = 0$ , (ii)  $\lim_{\tau_p \rightarrow 0} \hat{k}(\tau_p) / \tau_p = \infty$ , and (iii)  $\lim_{\tau_p \rightarrow 0} \hat{k}(\tau_p)^2 / \tau_p = 0$ .

**PROOF OF LEMMA 2** To see that  $\hat{k}(\tau_p)$  approaches zero as  $\tau_p$  approaches zero, first recall that  $\hat{k}(\tau_p)$  increases in  $\tau_p$ . Thus as  $\tau_p$  approaches zero,  $\hat{k}(\tau_p)$  has a limit that is less than infinity. Suppose that this limit is strictly positive. Then it is easy to see that for  $\tau_p$  small enough

$$\begin{aligned} 0 &= H(\hat{k}(\tau_p)) \\ &= \left( -\left( \frac{\tau_s}{\tau_p} \tau_h + \tau_p \right) \hat{k}(\tau_p)^3 + \tau_h \hat{k}(\tau_p)^2 \right) \tau_b + \tau_p \left( \hat{k}(\tau_p)^2 + 2\hat{k}(\tau_p) + 1 \right) (\tau_h - \tau_s \hat{k}(\tau_p)) < 0, \end{aligned}$$

which is a contradiction. Hence, we must have  $\lim_{\tau_p \rightarrow 0} \hat{k}(\tau_p) = 0$ .

By letting  $z(\tau_p) = \hat{k}(\tau_p) / \tau_p$ , Equation (3) can be rewritten as

$$\tau_p \left[ \left( -(\tau_s \tau_p \tau_h + \tau_p) z(\tau_p)^3 + \tau_h \tau_p z(\tau_p)^2 \right) \tau_b + \left( \tau_p^2 z(\tau_p)^2 + 2\tau_p z(\tau_p) + 1 \right) (\tau_h - \tau_s \tau_p z(\tau_p)) \right] = 0.$$

Thus for all  $\tau_p$  we must have

$$\left( -(\tau_s \tau_p \tau_h + \tau_p) z(\tau_p)^3 + \tau_h \tau_p z(\tau_p)^2 \right) \tau_b + \left( \tau_p^2 z(\tau_p)^2 + 2\tau_p z(\tau_p) + 1 \right) (\tau_h - \tau_s \tau_p z(\tau_p)) = 0.$$

Suppose  $\liminf_{\tau_p \rightarrow 0} z(\tau_p) < \infty$ . Then, there is clearly  $\tau_p$  small enough such that the left-hand side is positive, which is a contradiction. Hence,  $\lim_{\tau_p \rightarrow 0} \hat{k}(\tau_p) / \tau_p = \infty$ .

Finally, from Equation (3), we know that:

$$\frac{\hat{k}(\tau_p)^2}{\tau_p} = \frac{\tau_h \left( 2\hat{k}(\tau_p) + 1 \right) - \tau_s \hat{k}(\tau_p)}{\left( \frac{\tau_s \hat{k}(\tau_p)}{\tau_p} \tau_h + \tau_s \hat{k}(\tau_p) - \tau_h \right) \tau_b - \tau_h \tau_p + \tau_p \tau_s \left( \hat{k}(\tau_p) + 2 \right)}.$$

Since the right-hand side approaches zero as  $\tau_p$  approaches zero, we establish that

$$\lim_{\tau_p \rightarrow 0} \hat{k}(\tau_p)^2 / \tau_p = 0. \blacksquare$$

Using Lemma 2, we inspect Equation (8) and see that as  $\tau_p$  approaches zero,

$$\hat{s}(s_{pi}) = \hat{s}(0) = \sigma_s \Phi^{-1}(1 - c).$$

Moreover, inspecting (6), we see that at the limit:

$$T = \frac{\hat{s}(0) - \theta}{\sigma_s}.$$

Hence, in the limit, the central bank infers  $\theta$  perfectly from the attack and acts optimally.  $\blacksquare$

#### PROOF OF PROPOSITION 6:

Recall that  $\tau_T = \tau_p \left( 1 + \frac{1}{\hat{k}} \right)^2$ . Since  $\hat{k}$  decreases in  $\tau_b$  and  $\tau_s$  (Proposition 3), it follows immediately that  $\tau_T$  increases in  $\tau_b$  and  $\tau_s$ . We know that the probability of a policy mistake decreases in  $\tau_b$  and  $\tau_T$  (Proposition 4). It then follows immediately that this probability decreases in  $\tau_b$  and  $\tau_s$ .

We now need to show that  $\partial \tau_T / \partial \tau_p > 0$  if  $\tau_p > \bar{\tau}_p$  and  $\partial \tau_T / \partial \tau_p < 0$  if  $\tau_p < \bar{\tau}_p$ , where  $\bar{\tau}_p = \frac{1}{8} \frac{\tau_h^2 \tau_b}{(\tau_h + \tau_s)^2} \left( \sqrt{1 + \frac{16\tau_s}{\tau_b}} + \frac{16\tau_s^2}{\tau_h \tau_b} - 1 \right)$ . We can write  $\partial \tau_T / \partial \tau_p$  as follows:

$$\frac{\partial \tau_T}{\partial \tau_p} = \frac{(1 + \hat{k})}{\hat{k}^2} \left[ \left( 1 + \hat{k} \right) - 2\tau_p \frac{1}{\hat{k}} \frac{\partial \hat{k}}{\partial \tau_p} \right], \quad (\text{A1})$$

Taking the total derivative of Equation 3 with respect to  $\tau_p$ , we get:

$$\frac{\partial \hat{k}}{\partial \tau_p} = \frac{-\tau_s \tau_b \left( \frac{\tau_h}{\tau_p^2} \right) \hat{k}^3 - \left( \hat{k}^2 + 2\hat{k} + 1 \right) \left( \tau_h - \tau_s \hat{k} \right)}{-3\tau_s \tau_b \left( \frac{\tau_h}{\tau_p} + 1 \right) \hat{k}^2 + 2\tau_b \tau_h \hat{k} + \tau_p \left( 2\hat{k} + 2 \right) \left( \tau_h - \tau_s \hat{k} \right) - \tau_s \tau_p \left( \hat{k}^2 + 2\hat{k} + 1 \right)}.$$

Substituting  $\partial k / \partial \tau_p$  in Equation A1 we see that  $\partial \tau_T / \partial \tau_p > 0$  iff

$$\frac{-2\tau_s \tau_b \left( \frac{\tau_h}{\tau_p} \right) \hat{k}^3 - 2\tau_p \left( \hat{k}^2 + 2\hat{k} + 1 \right) \left( \tau_h - \tau_s \hat{k} \right)}{-3\tau_s \tau_b \left( \frac{\tau_h}{\tau_p} + 1 \right) \hat{k}^3 + 2\tau_b \tau_h \hat{k}^2 + \tau_p \left( 2\hat{k}^2 + 2\hat{k} \right) \left( \tau_h - \tau_s \hat{k} \right) - \tau_s \tau_p \left( \hat{k}^3 + 2\hat{k}^2 + \hat{k} \right)} < 1 + \hat{k}. \quad (\text{A2})$$

We use Equation 3 to obtain:

$$-2\tau_s \tau_b \left( \frac{\tau_h}{\tau_p} \right) \hat{k}^3 = 2\tau_s \tau_b \hat{k}^3 - 2\tau_b \tau_h \hat{k}^2 - 2\tau_p \left( \hat{k}^2 + 2\hat{k} + 1 \right) \left( \tau_h - \tau_s \hat{k} \right), \quad (\text{A3})$$

and

$$-3\tau_s \tau_b \left( \frac{\tau_h}{\tau_p} + 1 \right) \hat{k}^3 = -3\tau_b \tau_h^2 \hat{k} - 3\tau_p \left( \hat{k}^2 + 2\hat{k} + 1 \right) \left( \tau_h - \tau_s \hat{k} \right). \quad (\text{A4})$$

Substituting Equations A3 and A4 into Equation A2 we get:

$$\frac{2\tau_s \tau_b \hat{k}^3 - 2\tau_b \tau_h \hat{k}^2 - 4\tau_p \left( \hat{k}^2 + 2\hat{k} + 1 \right) \left( \tau_h - \tau_s \hat{k} \right)}{\left( -3\tau_b \tau_h^2 \hat{k} - 3\tau_p \left( \hat{k}^2 + 2\hat{k} + 1 \right) \left( \tau_h - \tau_s \hat{k} \right) + 2\tau_b \tau_h \hat{k}^2 \right) + \tau_p \left( 2\hat{k}^2 + 2\hat{k} \right) \left( \tau_h - \tau_s \hat{k} \right) - \tau_s \tau_p \left( \hat{k}^3 + 2\hat{k}^2 + \hat{k} \right)} < 1 + \hat{k}.$$

Simplifying the above inequality we obtain:

$$\frac{-2\tau_b \hat{k}^2 \left( \tau_h - \hat{k} \tau_s \right) - 4\tau_p \left( \hat{k}^2 + 2\hat{k} + 1 \right) \left( \tau_h - \tau_s \hat{k} \right)}{-\tau_b \tau_h \hat{k}^2 - 2\tau_p \left( 1 + \hat{k} \right) \left( \tau_h - \tau_s \hat{k} \right) - \tau_p \tau_h \left( 1 + \hat{k} \right)^2} < 1 + \hat{k}.$$

Since  $\hat{k} < \tau_h / \tau_s$  (Lemma 1) the denominator of the previous inequality is strictly negative.

Thus the above inequality holds iff

$$\begin{aligned} & -2\tau_b \hat{k}^2 \left( \tau_h - \hat{k} \tau_s \right) - 4\tau_p \left( \hat{k}^2 + 2\hat{k} + 1 \right) \left( \tau_h - \tau_s \hat{k} \right) + \tau_b \tau_h \hat{k}^2 \left( 1 + \hat{k} \right) + 2\tau_p \left( 1 + \hat{k} \right)^2 \left( \tau_h - \tau_s \hat{k} \right) \\ & + \tau_p \tau_h \left( 1 + \hat{k} \right)^3 > 0. \end{aligned}$$

or by simplifying iff

$$-2 \left( \tau_h - \hat{k} \tau_s \right) \left( \tau_p \left( 1 + \hat{k} \right)^2 + \tau_b \hat{k}^2 \right) + \tau_b \tau_h \hat{k}^2 \left( 1 + \hat{k} \right) + \tau_p \tau_h \left( 1 + \hat{k} \right)^3 > 0. \quad (\text{A5})$$

Using Equation 3 we get the following two equations:

$$-2\tau_p \left(1 + \hat{k}\right)^2 \left(\tau_h - \tau_s \hat{k}\right) = -2\tau_s \tau_b \left(\frac{\tau_h}{\tau_p} + 1\right) \hat{k}^3 + 2\tau_b \tau_h \hat{k}^2$$

and

$$\tau_b \tau_h \hat{k}^2 + \tau_p \left(1 + \hat{k}\right)^2 \tau_h = \tau_s \tau_b \left(\frac{\tau_h}{\tau_p} + 1\right) \hat{k}^3 + \tau_p \left(1 + \hat{k}\right)^2 \tau_s \hat{k}.$$

Substituting into Equation A5 we get:

$$-2\tau_s \tau_b \left(\frac{\tau_h}{\tau_p} + 1\right) \hat{k}^3 + 2\tau_b \tau_h \hat{k}^2 - 2\left(\tau_h - \hat{k}\tau_s\right) \tau_b \hat{k}^2 + \left(1 + \hat{k}\right) \left(\tau_s \tau_b \left(\frac{\tau_h}{\tau_p} + 1\right) \hat{k}^3 + \tau_p \left(1 + \hat{k}\right)^2 \tau_s \hat{k}\right) > 0.$$

Rewriting the above inequality we get:

$$\tau_s \tau_b \left(\frac{\tau_h}{\tau_p} + 1\right) \hat{k}^3 \left(\hat{k} - 1\right) + 2\tau_s \tau_b \hat{k}^3 + \tau_p \tau_s \left(1 + \hat{k}\right)^3 \hat{k} > 0.$$

Dividing by  $\hat{k}^2$  and recalling the definition of  $\tau_T$  we obtain:

$$\tau_b \left(\frac{\tau_h}{\tau_p} + 1\right) \left(\hat{k} - 1\right) + 2\tau_b + \tau_T \left(1 + \hat{k}\right) > 0$$

or

$$\hat{k} \left(1 + \frac{\tau_p}{\tau_h + \tau_p} \frac{\tau_T}{\tau_b}\right) > 1 - \frac{\tau_p}{\tau_h + \tau_p} \left(2 + \frac{\tau_T}{\tau_b}\right). \quad (\text{A6})$$

Using Equation 3 one more time we get:

$$\begin{aligned} -\tau_s \tau_b \left(\frac{\tau_h}{\tau_p} + 1\right) \hat{k} + \tau_b \tau_h + \tau_T \left(\tau_h - \tau_s \hat{k}\right) &= 0 \\ \Rightarrow \hat{k} &= \frac{\tau_h}{\tau_s} \frac{\frac{\tau_T}{\tau_b} + 1}{\frac{\tau_T}{\tau_b} + \left(\frac{\tau_h}{\tau_p} + 1\right)}. \end{aligned}$$

Substituting into Equation A6 and simplifying we get:

$$\tau_T > \tau_b \frac{\frac{\tau_s}{\tau_p} (\tau_h - \tau_p) - \tau_h}{\tau_h + \tau_s}. \quad (\text{A7})$$

If  $\tau_p < \tau_s \tau_h / (\tau_s + \tau_h)$  then the denominator above is negative and the inequality is trivially satisfied. Thus let's assume that  $\tau_p > \tau_s \tau_h / (\tau_s + \tau_h)$ . (When we derive the cutoff  $\bar{\tau}_p$ , we will need to check that it is indeed less than  $\tau_s \tau_h / (\tau_s + \tau_h)$ . Some tedious algebra shows that this is indeed the case.) Given this assumption we can rewrite the inequality in A7 as:

$$\frac{1}{\hat{k}} > \left( \left( \frac{\tau_b}{\tau_p} \frac{\frac{\tau_s}{\tau_p} (\tau_h - \tau_p) - \tau_h}{\tau_h + \tau_s} \right)^{0.5} - 1 \right). \quad (\text{A8})$$

Dividing Equation 3 by  $\hat{k}^3$  and letting  $w = 1/\hat{k}$  we can rewrite the equilibrium condition as:

$$-\tau_s \tau_b \left( \frac{\tau_h}{\tau_p} + 1 \right) + \tau_b \tau_h w + \tau_p (1 + w)^2 (\tau_h w - \tau_s) = 0. \quad (\text{A9})$$

Equation 3 has a unique root and so does Equation A9. Moreover, Equation A9 crosses zero from below. Let

$$\bar{w} = \left( \left( \frac{\tau_b \frac{\tau_s}{\tau_p} (\tau_h - \tau_p) - \tau_h}{\tau_p (\tau_h + \tau_s)} \right)^{0.5} - 1 \right).$$

If the left hand side of Equation A9 is negative when evaluated at  $\bar{w}$  then  $w = 1/\hat{k} > \bar{w}$ .

Substituting  $\bar{w}$  and simplifying this condition becomes:

$$-\tau_b \frac{\tau_h}{\tau_p} \frac{\tau_s}{\tau_h + \tau_s} \left( 2\tau_h + 2\tau_s - \tau_h \sqrt{-\frac{\tau_b}{\tau_p^2 (\tau_h + \tau_s)} (\tau_h \tau_p - \tau_h \tau_s + \tau_p \tau_s)} \right) < 0$$

which holds iff

$$4 \frac{(\tau_h + \tau_s)^3}{\tau_b \tau_h^2} \tau_p^2 + (\tau_h + \tau_s) \tau_p - \tau_h \tau_s > 0.$$

Solving the above quadratic equation for  $\tau_p$  we find that it is strictly positive iff  $\tau_p > \bar{\tau}_p$ .

Therefore  $\partial \tau_T / \partial \tau_p > 0$  iff  $\tau_p > \bar{\tau}_p$ . ■

## PROOF OF PROPOSITION 7:

Suppose that speculators follow linear strategies, that is, a speculator attacks if and only if his signal  $s_i$  is below  $s_c(0) - k_c s_{pi}$ . Here  $k_c$  is different than  $\hat{k}$  because speculators are best responding to the central bank that commits to overweighting its private signal compared to what would ex-post be optimal. The size of the attack from speculators given  $\theta$  and  $\epsilon_p$  is  $A(\theta, \epsilon_p) = \Phi \left( \frac{s_c(0) - k_c \sigma_p \epsilon_p - (1 + k_c) \theta}{\sqrt{\sigma_s^2 + k_c^2 \sigma_h^2}} \right)$ . The central bank observes  $T(\theta, \epsilon_p) = \Phi^{-1}(A)$  which can be written as:

$$\frac{s_c(0) - \sqrt{\sigma_s^2 + k_c^2 \sigma_h^2} T}{1 + k_c} = \theta + \frac{k_c \sigma_p}{1 + k_c} \epsilon_p.$$

Let  $\tau_T^c = \tau_p (1 + k_c)^2 / k_c^2$  be the precision of the attack under commitment. Suppose that the central bank abandons the status quo if and only if

$$\frac{\tau_T^c - \beta}{\tau_T^c + \tau_b} \left( \frac{s_c(0) - \sqrt{\sigma_s^2 + k_c^2 \sigma_h^2} T}{(1 + k_c)} \right) + \frac{\tau_b + \beta}{\tau_T^c + \tau_b} s_b \leq 0$$

or

$$T \geq \frac{s_c(0)}{\sqrt{\sigma_s^2 + k_c^2 \sigma_h^2}} + \frac{1 + k_c}{\sqrt{\sigma_s^2 + k_c^2 \sigma_h^2}} \frac{\tau_b + \beta}{\tau_T^c - \beta} s_b$$

for  $\tau_T^c > \beta > 0$ . Here,  $\beta$  is the deviation from ex-post optimal weights. It measures the increase (decrease) in the weight given to the central bank's private signal (to the information in the attack) relative to the ex-post optimal level. The posterior belief of the regime change for a speculator with signals  $s_i$  and  $s_{pi}$  is now expressed as follows:

$$\begin{aligned} & \Pr \left( T \geq \frac{s_c(0)}{\sqrt{\sigma_s^2 + k_c^2 \sigma_h^2}} + \frac{1 + k_c}{\sqrt{\sigma_s^2 + k_c^2 \sigma_h^2}} \frac{\tau_b + \beta}{\tau_T^c - \beta} s_b \mid s_i, s_{pi} \right) \\ &= \Pr \left( \left( 1 + (1 + k_c) \frac{\tau_b + \beta}{\tau_T^c - \beta} \right) \theta + (1 + k_c) \frac{\tau_b + \beta}{\tau_T^c - \beta} \sigma_b \epsilon_b - k_c \sigma_h \eta_i \leq -k_c s_{pi} \mid s_i, s_{pi} \right) \\ &= \Phi \left( \left( \begin{aligned} & - \left( (1 + (1 + k_c) \frac{\tau_b + \beta}{\tau_T^c - \beta}) \frac{\tau_h \tau_p}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s} + \frac{\tau_s \tau_h + \tau_p \tau_h}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s} k_c \right) s_{pi} \\ & - \left( k_c \frac{\tau_p \tau_s}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s} + (1 + (1 + k_c) \frac{\tau_b + \beta}{\tau_T^c - \beta}) \frac{\tau_s \tau_h + \tau_p \tau_s}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s} \right) s_i \end{aligned} \right) / \Omega^c \right) \end{aligned}$$

where  $\Omega^c$  is the variance of  $\left( 1 + (1 + k_c) \frac{\tau_b + \beta}{\tau_T^c - \beta} \right) \theta + (1 + k_c) \frac{\tau_b + \beta}{\tau_T^c - \beta} \sigma_b \epsilon_b - k_c \sigma_h \eta_i$ . Following similar steps to those in the proof of Proposition 1, we get that  $k_c$  must satisfy:

$$\begin{aligned} & - (1 + (1 + k_c) \frac{\tau_b + \beta}{\tau_T^c - \beta}) \frac{\tau_h \tau_p}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s} - \frac{\tau_s \tau_h + \tau_p \tau_h}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s} k_c \\ & + k_c^2 \frac{\tau_p \tau_s}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s} + k_c (1 + (1 + k_c) \frac{\tau_b + \beta}{\tau_T^c - \beta}) \frac{\tau_s \tau_h + \tau_p \tau_s}{\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s} = 0. \end{aligned}$$

Multiplying through with  $\tau_s \tau_h + \tau_p \tau_h + \tau_p \tau_s$ , substituting for  $\tau_T^c$  and rearranging the above equation we obtain:

$$- \tau_h k_c^3 \tau_s (\tau_b + \beta) + \tau_p (\tau_h - k_c \tau_s) \tau_b k_c^2 + \tau_p^2 (\tau_h - k_c \tau_s) (k_c + 1)^2 = 0. \quad (\text{A10})$$

For  $\beta = 0$ , Equation A10 is the same as Equation 3. Since Equation 3 has a unique positive root, Equation A10 also has a unique positive root for  $\beta$  small enough. By Lemma 1,  $\hat{k} < \tau_h / \tau_s$ . Therefore, for  $\beta$  small enough,  $k_c < \tau_h / \tau_s$ . Taking total derivative of Equation A10 with respect to  $\beta$  we obtain:

$$\frac{\partial k_c}{\partial \beta} = \frac{\tau_h \tau_s k_c^3}{-3 \tau_h \tau_s (\tau_b + \beta) k_c^2 + 2 \tau_p \tau_b (\tau_h - k_c \tau_s) k_c + 2 \tau_p^2 (\tau_h - k_c \tau_s) (k_c + 1) - \tau_p^2 \tau_s (k_c + 1)^2}.$$

Using Equation A10 we can rewrite this as:

$$\frac{\partial k_c}{\partial \beta} = \frac{\tau_h \tau_s k_c^4}{- \tau_p \tau_b (\tau_h - k_c \tau_s) k_c^2 - \tau_p^2 (\tau_h - k_c \tau_s) (k_c + 1) (k_c + 3) - \tau_p^2 \tau_s k_c (k_c + 1)^2} < 0$$

where the inequality follows from  $k_c < \tau_h / \tau_s$ . Hence, given that  $\tau_T^c = \tau_p (1 + k_c)^2 / k_c^2$ , increasing  $\beta$  always leads to a more informative attack.

Next, we compute the ex-ante probability of abandoning the status quo for a given  $\theta$  :

$$\begin{aligned} & \Pr \left( T \geq \frac{s_c(0)}{\sqrt{\sigma_s^2 + k_c^2 \sigma_h^2}} + \frac{1 + k_c}{\sqrt{\sigma_s^2 + k_c^2 \sigma_h^2}} \frac{\tau_b + \beta}{\tau_T^c - \beta} s_b | \theta \right) \\ &= \Phi \left( -\frac{1}{\sigma_b} \frac{\left(1 + \frac{\tau_T^c - \beta}{\tau_b + \beta}\right)}{\sqrt{1 + \frac{\tau_b}{\tau_T^c} \frac{(\tau_T^c - \beta)^2}{(\tau_b + \beta)^2}}} \theta \right) = \Phi \left( -\frac{1}{\sigma_b} \left( \frac{\tau_b \tau_T^c + (\tau_T^c)^2}{\tau_b \tau_T^c + \beta^2} \right)^{\frac{1}{2}} \theta \right). \end{aligned}$$

Moreover,

$$\begin{aligned} \frac{\partial}{\partial \beta} \left( \frac{\tau_b \tau_T^c + (\tau_T^c)^2}{\tau_b \tau_T^c + \beta^2} \right) &= \frac{\tau_b \frac{\partial \tau_T^c}{\partial \beta} + 2 \tau_T^c \frac{\partial \tau_T^c}{\partial \beta}}{\tau_b \tau_T^c + \beta^2} - \frac{\tau_b \tau_T^c + (\tau_T^c)^2}{(\tau_b \tau_T^c + \beta^2)^2} \left( \tau_b \frac{\partial \tau_T^c}{\partial \beta} + 2 \beta \right) \\ &= \frac{1}{\tau_b \tau_T^c + \beta^2} \left( \left( \frac{\tau_b (\tau_T^c)^2 + 2 \tau_T^c \beta^2 + \tau_b \beta^2}{\tau_b \tau_T^c + \beta^2} \right) \frac{\partial \tau_T^c}{\partial \beta} - \frac{\tau_b \tau_T^c + (\tau_T^c)^2}{(\tau_b \tau_T^c + \beta^2)} 2 \beta \right). \end{aligned}$$

Computing at  $\beta = 0$ ,

$$\left. \frac{\partial}{\partial \beta} \left( \frac{\tau_b \tau_T^c + (\tau_T^c)^2}{\tau_b \tau_T^c + \beta^2} \right) \right|_{\beta=0} = \frac{1}{\tau_b} \left. \frac{\partial \tau_T^c}{\partial \beta} \right|_{\beta=0} > 0.$$

Thus, for  $\theta > 0$  ( $\theta < 0$ ) the ex-ante probability of abandoning the status quo decreases (increases) if the central bank sets  $\beta$  slightly larger than zero. ■

#### PROOF OF PROPOSITION 8:

The central bank observes both  $s_b$  and  $s_a$ , and speculators observe  $s_a$ ,  $s_{pi}$  and  $s_i$ . We construct a linear equilibrium in which a speculator attacks if and only if  $s_i \leq s_{tr}(0) - m s_a - k_{tr} s_{pi}$  where  $m > 0$ ,  $k_{tr} > 0$ . The size of the attack from speculators given  $\theta$ ,  $\epsilon_p$  and  $s_a$  is  $A(\theta, \epsilon_p, s_a) = \Phi \left( \frac{s_{tr}(0) - m s_a - k_{tr} \sigma_p \epsilon_p - (1 + k_{tr}) \theta}{\sqrt{\sigma_s^2 + k_{tr}^2 \sigma_h^2}} \right)$ . The central bank observes  $T(\theta, s_p) = \Phi^{-1}(A)$ , or equivalently, it observes

$$T = \frac{s_{tr}(0) - m s_a - k_{tr} \sigma_p \epsilon_p - (1 + k_{tr}) \theta}{\sqrt{\sigma_s^2 + k_{tr}^2 \sigma_h^2}}$$

which can be rewritten as:

$$\frac{s_{tr}(0) - m s_a - \sqrt{\sigma_s^2 + k_{tr}^2 \sigma_h^2} T}{1 + k_{tr}} = \theta + \frac{k_{tr} \sigma_p \epsilon_p}{1 + k_{tr}}.$$

Thus, the precision of the attack as a signal of the fundamental is

$$\tau_T = \frac{\tau_p (1 + k_{tr})^2}{k_{tr}^2},$$

and

$$E[\theta|T, s_b, \epsilon_a] = \frac{\tau_T}{\tau_T + \tau_b} \left( \frac{s_{tr}(0) - m(s_b + \sigma_a \epsilon_a) - \sqrt{\sigma_s^2 + k_{tr}^2 \sigma_h^2} T}{1 + k_{tr}} \right) + \frac{\tau_b}{\tau_T + \tau_b} s_b.$$

This implies the status quo is abandoned if and only if

$$T \geq \frac{s_{tr}(0)}{\sqrt{\sigma_s^2 + k_{tr}^2 \sigma_h^2}} - \frac{m s_a}{\sqrt{\sigma_s^2 + k_{tr}^2 \sigma_h^2}} + \frac{(1 + k_{tr}) \frac{\tau_b}{\tau_T}}{\sqrt{\sigma_s^2 + k_{tr}^2 \sigma_h^2}} s_b.$$

Let

$$\tau_a = \frac{1}{\sigma_a^2}, \quad \tau'_a = \frac{\tau_a \tau_b}{\tau_a + \tau_b} \quad \text{and} \quad \tau'_p = \frac{\tau_h \tau_p}{\tau_h + \tau_p}.$$

For a speculator,  $\theta$  is distributed with mean  $\frac{\tau_s}{\tau_s + \tau'_p + \tau'_a} s_i + \frac{\tau'_p}{\tau_s + \tau'_p + \tau'_a} s_{pi} + \frac{\tau'_a}{\tau_s + \tau'_p + \tau'_a} s_a$ , and  $\sigma_h \eta_i$  is distributed with mean  $\frac{\tau'_p}{\tau_h} \frac{\tau_s(s_{pi} - s_i) + (s_{pi} - s_a)\tau'_a}{\tau'_p + \tau'_a + \tau_s}$ , and  $\sigma_a \epsilon_a$  is distributed with mean  $\frac{\tau'_a}{\tau_a} \frac{\tau_s(s_a - s_i) + (s_a - s_{pi})\tau'_p}{\tau'_p + \tau'_a + \tau_s}$ . Let  $\Omega$  be the standard deviation of  $\theta - \frac{\tau_b}{\tau_T} (1 + k_{tr}) \sigma_a \epsilon_a - k_{tr} \sigma_h \eta_i$ . The posterior belief of the regime change for a speculator with signals  $s_i$ ,  $s_{pi}$  and  $s_a$  is expressed as follows:

$$\begin{aligned} & \Pr \left( T \geq \frac{s_{tr}(0)}{\sqrt{\sigma_s^2 + k_{tr}^2 \sigma_h^2}} - \frac{m s_a}{\sqrt{\sigma_s^2 + k_{tr}^2 \sigma_h^2}} + \frac{(1 + k_{tr}) \frac{\tau_b}{\tau_T}}{\sqrt{\sigma_s^2 + k_{tr}^2 \sigma_h^2}} s_b | s_i, s_{pi}, s_a \right) \\ &= \Pr \left( \theta - \frac{\tau_b}{\tau_T} (1 + k_{tr}) \sigma_a \epsilon_a - k_{tr} \sigma_h \eta_i \leq -\frac{\tau_b}{\tau_T} (1 + k_{tr}) s_a - k_{tr} s_{pi} | s_i, s_{pi}, s_a \right) \\ &= \Phi \left( \left( \begin{aligned} & -\frac{\tau_b}{\tau_T} (1 + k_{tr}) s_a - k_{tr} s_{pi} - \left( \frac{\tau_s}{\tau_s + \tau'_p + \tau'_a} s_i + \frac{\tau'_p}{\tau_s + \tau'_p + \tau'_a} s_{pi} + \frac{\tau'_a}{\tau_s + \tau'_p + \tau'_a} s_a \right) \\ & + \frac{\tau_b}{\tau_T} (1 + k_{tr}) \left( \frac{\tau'_a}{\tau_a} \frac{\tau_s(s_a - s_i) + (s_a - s_{pi})\tau'_p}{\tau'_p + \tau'_a + \tau_s} \right) + k_{tr} \frac{\tau'_p}{\tau_h} \frac{\tau_s(s_{pi} - s_i) + (s_{pi} - s_a)\tau'_a}{\tau'_p + \tau'_a + \tau_s} \end{aligned} \right) / \Omega \right). \end{aligned}$$

The agent with signal  $s_{tr}(0) - m s_a - k_{tr} s_{pi}$  must be indifferent between attacking or not:

$$c = \Phi \left( \left( \begin{aligned} & -\frac{\tau_b}{\tau_T} (1 + k_{tr}) s_a - k_{tr} s_{pi} - \left( \frac{\tau_s}{\tau_s + \tau'_p + \tau'_a} (s_{tr}(0) - m s_a - k_{tr} s_{pi}) + \frac{\tau'_p}{\tau_s + \tau'_p + \tau'_a} s_{pi} + \frac{\tau'_a}{\tau_s + \tau'_p + \tau'_a} s_a \right) \\ & + \frac{\tau_b}{\tau_T} (1 + k_{tr}) \left( \frac{\tau'_a}{\tau_a} \frac{\tau_s(s_a - (s_{tr}(0) - m s_a - k_{tr} s_{pi})) + (s_a - s_{pi})\tau'_p}{\tau'_p + \tau'_a + \tau_s} \right) + k_{tr} \frac{\tau'_p}{\tau_h} \frac{\tau_s(s_{pi} - (s_{tr}(0) - m s_a - k_{tr} s_{pi})) + (s_{pi} - s_a)\tau'_a}{\tau'_p + \tau'_a + \tau_s} \end{aligned} \right) / \Omega \right)$$

In a linear equilibrium the coefficients on  $s_{pi}$  and  $s_a$  must both be zero. Setting the coefficient of  $s_{pi}$  to zero we see that  $k_{tr}$  must satisfy:

$$-k_{tr} + \frac{\tau_s}{\tau_s + \tau'_p + \tau'_a} k_{tr} - \frac{\tau'_p}{\tau_s + \tau'_p + \tau'_a} + \frac{\tau_b}{\tau_T} (1 + k_{tr}) \frac{\tau'_a}{\tau_a} \frac{\tau_s k_{tr} - \tau'_p}{\tau'_p + \tau'_a + \tau_s} + k_{tr} \frac{\tau'_p}{\tau_h} \frac{\tau_s (1 + k_{tr}) + \tau'_a}{\tau'_p + \tau'_a + \tau_s} = 0, \quad (\text{A11})$$

and setting the coefficient of  $s_a$  to zero we see that, given  $k_{tr}$ ,  $m$  must satisfy:

$$-\frac{\tau_b}{\tau_T} (1 + k_{tr}) + \frac{\tau_s}{\tau_s + \tau'_p + \tau'_a} m - \frac{\tau'_a}{\tau_s + \tau'_p + \tau'_a} + \frac{\tau_b}{\tau_T} (1 + k_{tr}) \frac{\tau'_a}{\tau_a} \frac{\tau_s (1 + m) + \tau'_p}{\tau'_p + \tau'_a + \tau_s} + k_{tr} \frac{\tau'_p}{\tau_h} \frac{\tau_s m - \tau'_a}{\tau'_p + \tau'_a + \tau_s} = 0. \quad (\text{A12})$$

Plugging in for  $\tau_T$ ,  $\tau'_p$  and  $\tau'_a$ , multiplying through with  $(\tau_p + \tau_h)(\tau_a + \tau_b)$  and simplifying Equation A11, we see that  $k_{tr}$  must be the root of:

$$H_{tr}(k) = \left( - \left( \frac{\tau_s}{\tau_p} \right) (\tau_h + \tau_p) k^3 + \tau_h k^2 \right) \tau_b + \tau_p (k+1)^2 (\tau_h - \tau_s k) + \frac{\tau_a}{\tau_a + \tau_b} \frac{\tau_b k}{\tau_p} (k^2 \tau_s (\tau_h + \tau_p) + \tau_h \tau_p). \quad (\text{A13})$$

Note that for  $\tau_a = 0$ ,  $H_{tr}(k)$  simplifies to  $H(k) = 0$ . ( $H(k)$  is defined in the proof of Proposition 1.) Since  $H(k) = 0$  has a unique strictly positive solution, for  $\tau_a$  small enough,  $H_{tr}(k) = 0$  also has a unique strictly positive solution. This immediately implies that  $m$  is also unique. Solving  $m$  as a function of  $k_{tr}$  we can verify that  $m > 0$ . This proves that there is a unique linear threshold equilibrium if  $\sigma_a$  is not too small.

Since  $\tau_a / (\tau_a + \tau_b)$  increases in  $\tau_a$ ,  $H_{tr}(k)$  shifts up for  $k > 0$  as  $\tau_a$  increases. Therefore  $k_{tr}$  increases in  $\tau_a$ . Thus  $\tau_T = \tau_p (1 + k_{tr})^2 / k_{tr}^2$  decreases in  $\tau_a$ . We can easily verify that the probability of a policy mistake given  $\theta$  is still  $\Phi(-\sqrt{\tau_b + \tau_T}\theta)$  for  $\theta > 0$  and  $1 - \Phi(-\sqrt{\tau_b + \tau_T}\theta)$  for  $\theta < 0$ . This proves the rest of the proposition. ■

## Appendix B

### Common Signal about Noise Trading

We now demonstrate that the central bank's policy decision is less efficient than the benchmark case even when speculators receive a common signal that is not about the fundamental. To illustrate this point suppose that there is some noise demand for the currency. Specifically, we assume that the central bank observes a noisy signal of the size of the attack from informed speculators,  $T = \Phi^{-1}(A) + \sigma_m \epsilon_m$ , where  $A$  is the size of the aggregate attack from speculators, and the noise component of this signal,  $\sigma_m \epsilon_m$ , is normally distributed with a mean of zero and standard deviation  $\sigma_m$ .<sup>18</sup>

Speculators' strategies are now functions of their private signals and the commonly observed noise level. Otherwise, equilibrium is defined analogously to the one in Definition 1. In this case, it is possible for speculators to coordinate on the level of noise demand and "fool" the central bank. The assumption on speculators' informational advantage about the noise trading level is motivated by the fact that although individual speculators may not know about the fundamental more than the central bank, they understand the institutional details or the workings of the currency market better. Under this assumption, we show that

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<sup>18</sup>This specification is introduced by Dasgupta (2007).

currency speculators trade more aggressively on their information when the noise component of currency attack is of a high level. As a result, the central bank cannot differentiate between a high noise attack or a low fundamental, which implies there might be occasions where the central bank on average abandons the status quo too often. We first state the result under these circumstances.

**Proposition B1** *There is a unique equilibrium where the speculators' threshold strategies are linear in  $\epsilon_m$  such that*

$$g(s) = \begin{cases} 1 & \text{if } s_i \leq \bar{s}(\epsilon_m) \\ 0 & \text{if } s_i > \bar{s}(\epsilon_m) \end{cases}, \quad (\text{B1})$$

and  $\delta(T, s_b) = \begin{cases} 1 & \text{if } T \leq \bar{T}(s_b) \\ 0 & \text{if } T > \bar{T}(s_b) \end{cases}$ , where  $\bar{s}(\epsilon_m) = \bar{s}(0) + \bar{k}\epsilon_m$  where  $\bar{k} > 0$  is the unique real root of the cubic equation:

$$\bar{k}^3 + 2\sigma_s\sigma_m\bar{k}^2 + (\sigma_s\sigma_m)^2\bar{k} - \sigma_b^2\sigma_s\sigma_m = 0.$$

$\bar{s}(0)$  satisfies

$$c = \Phi \left( \frac{-\left(1 + \frac{\tau_b}{\bar{\tau}_T}\right) \bar{s}(0)}{\sqrt{\left(1 + \frac{\tau_b}{\bar{\tau}_T}\right)^2 \sigma_s^2 + \left(\frac{\tau_b}{\bar{\tau}_T}\right)^2 \sigma_b^2}} \right), \quad (\text{B2})$$

and

$$\bar{T}(s_b) = \frac{1}{\sigma_s} \left[ \bar{s}(0) + \frac{(\bar{k} + \sigma_s\sigma_m)^2}{\sigma_b^2} s_b \right], \quad (\text{B3})$$

where  $\bar{\tau}_T = \frac{1}{(\bar{k} + \sigma_s\sigma_m)^2}$  is the precision of the attack as a signal of the fundamental.

PROOF OF PROPOSITION B1:

Suppose an agent attacks if and only if  $s_i \leq \bar{s}(\epsilon_m) = \bar{s}(0) + \bar{k}\epsilon_m$ . The size of the attack from speculators is  $A(\theta, \epsilon_m) = \Phi \left( \frac{\bar{s}(\epsilon_m) - \theta}{\sigma_s} \right)$ . The central bank observes  $T(\theta, \epsilon_m) = \Phi^{-1}(A) + \sigma_m\epsilon_m$ , or equivalently, it observes

$$\begin{aligned} T &= \frac{\bar{s}(0) + \bar{k}\epsilon_m - \theta}{\sigma_s} + \sigma_m\epsilon_m, \\ \bar{s}(0) - \sigma_s T &= \theta - (\bar{k} + \sigma_s\sigma_m)\epsilon_m, \end{aligned}$$

and

$$E[\theta|T, s_b] = \frac{\bar{\tau}_T}{\bar{\tau}_T + \tau_b} (\bar{s}(0) - \sigma_s T) + \frac{\tau_b}{\bar{\tau}_T + \tau_b} s_b, \quad (\text{B4})$$

where  $\bar{\tau}_T = \frac{1}{(\bar{k} + \sigma_s \sigma_m)^2}$ . This implies the status quo is abandoned if and only if

$$T \geq \frac{1}{\sigma_s} \left[ \bar{s}(0) + \frac{\tau_b}{\bar{\tau}_T} s_b \right] = \bar{T}(s_b),$$

which is Equation (B3).

The posterior belief of the regime change for a speculator with signal  $s_i$  and  $\epsilon_m$  is expressed as follows:

$$\Pr \left( T \geq \frac{1}{\sigma_s} \left[ \bar{s}(0) + \frac{\tau_b}{\bar{\tau}_T} s_b \right] \mid s_i, \epsilon_m \right) = \Phi \left( \frac{(\bar{k} + \sigma_s \sigma_m) \epsilon_m - \left( 1 + \frac{\tau_b}{\bar{\tau}_T} \right) s_i}{\sqrt{\left( 1 + \frac{\tau_b}{\bar{\tau}_T} \right)^2 \sigma_s^2 + \left( \frac{\tau_b}{\bar{\tau}_T} \right)^2 \sigma_b^2}} \right).$$

Hence,  $\bar{s}(\epsilon_m) = \bar{s}(0) + k\epsilon_m$  must solve

$$c = \Phi \left( \frac{-\left( 1 + \frac{\tau_b}{\bar{\tau}_T} \right) \bar{s}(0) + \left( \sigma_s \sigma_m - \bar{k} \frac{\tau_b}{\bar{\tau}_T} \right) \epsilon_m}{\sqrt{\left( 1 + \frac{\tau_b}{\bar{\tau}_T} \right)^2 \sigma_s^2 + \left( \frac{\tau_b}{\bar{\tau}_T} \right)^2 \sigma_b^2}} \right). \quad (\text{B5})$$

To solve this equation for all  $\epsilon_m$ , the coefficient of  $\epsilon_m$  must be zero. In other words,

$$\sigma_s \sigma_m - \bar{k} \frac{\tau_b}{\bar{\tau}_T} = 0.$$

Rearranging we find that  $\bar{k}$  must solve:

$$\bar{k}^3 + 2\sigma_s \sigma_m \bar{k}^2 + (\sigma_s \sigma_m)^2 \bar{k} - \sigma_b^2 \sigma_s \sigma_m = 0. \quad (\text{B6})$$

To see that this equation has a unique real root we compute the discriminant  $\Delta$  of the cubic equation:

$$\Delta = 4(\sigma_s \sigma_m)^4 \frac{1}{\tau_b} + 27 \left( \frac{\sigma_s \sigma_m}{\tau_b} \right)^2 > 0.$$

Since  $\Delta > 0$  the equation has a unique real root. Moreover, the left-hand side of equation (B6) goes to  $-\infty$  as  $\bar{k}$  goes to  $-\infty$  and it is negative at  $\bar{k} = 0$ . Since the equation has a single real root, it must cross zero at some  $\bar{k} > 0$ . Given  $\bar{k}$ , we obtain  $\bar{s}(0)$  as the solution to Equation (B2). ■

In other words, the equilibrium strategy for the central bank is to abandon the exchange regime if and only if the observed signal of aggregate attack,  $T$ , is greater than or equal to the threshold,  $\bar{T}(s_b)$ , a function of the central bank's private signal,  $s_b$ . The equilibrium strategy for a speculator who receives a signal,  $s$ , is to attack if and only if  $s$  falls below a threshold value,  $\bar{s}(\epsilon_m)$ , which is a linear increasing function of the noise trading level in the

currency market.

**Proposition B2** *The weight  $\bar{k}$  put by speculators on  $\epsilon_m$  in the unique linear threshold equilibrium characterized by Proposition B1 is greater than the weight  $\bar{k}_{BM}$  that would be put on  $\epsilon_m$  in a game where the central bank does not attempt to get information about  $\theta$  from the size of the attack.*

The proof is straightforward since  $\bar{k}$  is strictly positive and if the central bank does not infer the fundamental from  $T$ ,  $\bar{k}_{BM} = 0$ .

## Appendix C

### Speculators Observe Fundamental Perfectly

We begin by defining equilibrium in this setting. Let  $A(\theta)$  denote the size of the aggregate attack given  $\theta$ ,  $g(\theta)$  the action of an agent given  $\theta$ , and  $\delta(A, s_b)$  the action of the central bank as a function of the size of the attack and its signal. Furthermore, let  $\nu(\theta|A, s_b)$  denote the posterior belief by the central bank conditional on  $A$  and  $s_b$ .

**Definition C1** *An equilibrium consists of a mapping  $A$  from the fundamental to the size of the attack, a strategy for the central bank,  $\delta(A, s_b)$ , a symmetric strategy for the agents,  $g(\theta)$  and a probability measure,  $\nu(\cdot|A, s_b)$ , such that*

$$\begin{aligned} \delta(A, s_b) &\in \operatorname{argmax}_{\delta \in \{0,1\}} \int_{-\infty}^{\infty} \delta \theta d\nu(\theta|A, s_b), \\ g(\theta) &\in \operatorname{argmax}_{a \in \{0,1\}} a \cdot \left[ \int_{-\infty}^{\infty} \mathbf{1}_{[\delta(A(\theta), \theta + \sigma_b \epsilon_b) = 0]} d\phi(\epsilon_b) - c \right] \\ \nu(\theta|A, s_b) &\text{ is obtained using Bayes' rule for any } A, s_b, \\ A(\theta) &= 1 \text{ if } g(\theta) = 1 \text{ and } A(\theta) = 0 \text{ otherwise.} \end{aligned}$$

Since speculators have information that is valuable to the central bank, in equilibrium, the central bank would make its policy decision based on the information revealed through speculators' actions as well as its own noisy private signal. When the fundamental is common knowledge among them, speculators may coordinate in various ways leading to different inferences by the central bank, resulting in multiple equilibria. In particular, speculators may be able to convince the central bank to abandon the fixed exchange rate regime even when it is not optimal for the central bank to do so. Our next proposition shows that multiplicity may arise when the speculators follow symmetric cut-off strategies.

**Proposition C1** *There exists  $\theta^* > 0$  such that for all  $\bar{\theta} \in [0, \theta^*]$  there is an equilibrium where  $g(\theta) = 1$  if  $\theta \leq \bar{\theta}$  and  $g(\theta) = 0$  if  $\theta > \bar{\theta}$ .*

PROOF OF PROPOSITION C1:

Suppose  $g(\theta) = 1$  if  $\theta \leq \bar{\theta}$  and  $g(\theta) = 0$  if  $\theta > \bar{\theta}$ . Hence, the expectation of  $\theta$  conditional on observing  $A = 1$  and  $s_b$  is

$$E[\theta | \theta \leq \bar{\theta}, s_b] = \frac{\int_{-\infty}^{\bar{\theta}} \theta \phi\left(\frac{\theta - s_b}{\sigma_b}\right) d\theta}{\Phi\left(\frac{\bar{\theta} - s_b}{\sigma_b}\right)}.$$

Next, we show that  $E[\theta | \theta \leq \bar{\theta}, s_b]$  is increasing in  $s_b$ . To see this, note that  $E[\theta | \theta \leq \bar{\theta}, s_b + \Delta] = \Delta - E[\theta | \theta \leq \bar{\theta} - \Delta, s_b]$ . Subtracting  $E[\theta | \theta \leq \bar{\theta}, s_b]$  from both sides, dividing by  $\Delta$  and letting  $\Delta$  go to zero, we obtain:

$$\frac{\partial E[\theta | \theta \leq \bar{\theta}, s_b]}{\partial s_b} = 1 - \frac{\partial E[\theta | \theta \leq \bar{\theta}, s_b]}{\partial \bar{\theta}}.$$

By Proposition 1 in Burdett (1996), we know that  $\frac{\partial E[\theta | \theta \leq \bar{\theta}, s_b]}{\partial \bar{\theta}} \in [0, 1]$ . Thus,  $\frac{\partial E[\theta | \theta \leq \bar{\theta}, s_b]}{\partial s_b} \geq 0$ .

Suppose  $\bar{\theta} \geq 0$ . Since  $\frac{\partial E[\theta | \theta \leq \bar{\theta}, s_b]}{\partial s_b} \geq 0$ , there exists a  $\hat{s}_b(\bar{\theta}) > 0$  (possibly infinite) such that

$$E[\theta | \theta \leq \bar{\theta}, s_b] = \begin{cases} \geq 0 & \text{if } s_b \geq \hat{s}_b(\bar{\theta}) \\ < 0 & \text{if } s_b < \hat{s}_b(\bar{\theta}) \end{cases}. \quad (\text{C1})$$

Therefore, the central bank's strategy is

$$\delta(A, s_b) = \begin{cases} 1 & \text{if } A = 1 \text{ and } s_b > \hat{s}_b(\bar{\theta}) \text{ or } A = 0 \\ 0 & \text{o.w.} \end{cases}. \quad (\text{C2})$$

Moreover,  $\hat{s}_b(\bar{\theta})$  is decreasing in  $\bar{\theta}$ .

Given the central bank's strategy and given the strategy of the other speculators, it is optimal for any speculator to set  $g(\theta) = 0$  if  $\theta > \bar{\theta}$ . Now, fix some  $\theta \leq \bar{\theta}$ . We want to show that if  $\bar{\theta} < \theta^*$  for some  $\theta^* > 0$  then it is optimal for the speculators to set  $g(\theta) = 1$ . Since in this case,  $A = 1$  and the probability that the central bank abandons the regime is  $\Phi\left(\frac{\hat{s}_b(\bar{\theta}) - \theta}{\sigma_b}\right)$ . Note that this probability is decreasing in  $\theta$ , so if it is optimal for speculators to attack at  $\bar{\theta}$ , it is also optimal to attack at all smaller  $\theta$ . Now note that  $\Phi\left(\frac{\hat{s}_b(\bar{\theta}) - \bar{\theta}}{\sigma_b}\right)$  is one at  $\bar{\theta} = 0$ , is continuous and decreases as  $\bar{\theta}$  increases. Therefore, there is a threshold  $\theta^* > 0$  such that  $\Phi\left(\frac{\hat{s}_b(\bar{\theta}) - \bar{\theta}}{\sigma_b}\right) - c \geq 0$  if  $\bar{\theta} \leq \theta^*$ . Thus it is optimal to attack for the speculators if  $\theta \leq \bar{\theta}$  where  $0 \leq \bar{\theta} \leq \theta^*$ . This proves the proposition. ■

In other words, this proposition shows that there are multiple equilibria in which the speculators follow a cut-off strategy: They attack the currency regime when their signal is below  $\bar{\theta}$  and do not attack otherwise. The cut-off value could be any  $\theta$  between  $[0, \theta^*]$ . This means that the central bank may devalue when the fundamental,  $\theta$ , is positive, which is not first-best optimal for the central bank.